A book, a pen, and the Sphere

Citation for published version:
Oosterhoff, RJ 2015, A book, a pen, and the *Sphere*: Reading Sacrobosco in the Renaissance. in M Feingold (ed.), *History of Universities.* vol. XXVIII/2, Oxford University Press, Oxford, pp. 1-54. https://doi.org/10.1093/acprof:oso/9780198743651.003.0001

Digital Object Identifier (DOI):
10.1093/acprof:oso/9780198743651.003.0001

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
History of Universities

A Book, a Pen, and the *Sphere*: Reading Sacrobosco in the Renaissance

Richard J. Oosterhoff[1]

Introduction

If you studied at university or grammar school in the sixteenth century, chances are you read the *Sphere* by John of Sacrobosco, the most popular introduction to the basics of astronomy since the thirteenth century.[2] Perhaps you owned one of the hundreds of cheap quartos of the book printed between the fifteenth and seventeenth centuries.[3] Or maybe you borrowed a copy from your college library, possibly one of the large folio editions that included the commentaries of medieval scholastic doctors like Robert Grosseteste or Pierre d'Ailly, alongside the orations and additions of moderns teaching at the universities of Padua or Paris. Perhaps you bought yourself a large volume, and instead of wasting space in your commonplace book you filled the margins of your text with observations and definitions, flagging contentious arguments raised by your professor in his mathematics lectures. In preparation for disputations you underlined and repeated keywords in the text, to review quickly as you moved on to more advanced treatises such as the *Theorics of the Planetary Motions*. However you read it, the *Sphere* summed up your knowledge of the cosmos.

The *Sphere* has received only passing attention from historians because it was so basic. The first book surveyed the qualities of a sphere, and how the universe was composed of nested spheres, with the four elements of the earth at the center. The second listed the various circles used by astronomers to delineate parts of the sphere, such as the zodiac, and the five zones of the earthly sphere. In the third book, the student would find a basic account

---

[1] richard.oosterhoff@gmail.com. I am grateful for the help of several modern readers of Sacrobosco: to Kate Isard, for stimulating questions; to Owen Gingerich, for sharing his collection; to Roger Gaskell, for sharing his vast knowledge and some images; to Alexander Marr, for responding to an early draft; to Kathleen Crowther and Peter Barker, for the type-script of a paper just published in *Isis*; and as always to Robert Goulding, for encouragement and close reading. The reviewers' comments were of great help in revising the article, particularly Renée Raphael, who generously gave up anonymity. The following abbreviations are used in this article: BL = British Library; BHS = Bibliothèque humaniste de Sélestat; BnF = Bibliothèque nationale de France; Houghton = Houghton Library, Harvard; Huntington = Huntington Library, Pasadena; Newberry = Newberry Library, Chicago.

[2] Sacrobosco is unknown except through his works, which include an *Algorismus* (basic arithmetic), *Computus*, as well as the *Sphere*. He possibly wrote a treatise on the quadrant as well. Lynn Thorndike, who wrote the one extensive study of Sacrobosco's work, suggests he was active in the first decades of the thirteenth century. Lynn Thorndike (ed. and trans.), *The Sphere of Sacrobosco and Its Commentators* (Chicago, 1949), 5–6.

[3] Jürgen Hamel, 'Johannes Sacrobosco: Handbuch der Astronomie, Kommenierte Bibliographie der Drucke der 'Sphaera' 1472- bis 1656', in Dietmar Fürst, Dieter B. Herrmann, and Eckehard Rothenberg (eds.), *Wege der Erkenntnis: Festschrift für Dieter B. Herrmann zum 65. Geburtstag* (Frankfurt am Main, 2004), 115–70.

of how the signs of the zodiac moved through the night sky, what this meant for days, seasons, and the various climates of the earth, from the inhospitably cold north to below the equator, which 'since living conditions are bad, is not reckoned as a clime'.[4] The fourth and last book provided a rough description of the sun's motion, with just enough theory of general planetary motion to explain eclipses.

The *Sphere* hardly represents bold and exciting science, even by the standards of the time. Yet its pages and the traces of its readers reveal something we miss when we focus on extraordinary works and outstanding readers such as Copernicus, Jean Bodin, or Galileo.[5] Indeed, as Mordechai Fiengold argued for the English universities and recently William Sherman for English literature, our perspective on exceptional works and thinkers risks distortion if we cannot set them against a background of 'ordinary' textbooks and their readers.[6] While examining the mathematical textbooks of Jacques Lefèvre d'Étaples and his Paris circle of students, I searched for such a backdrop against which to measure these works and their uses; finding none, I began to compare all the copies of such textbooks that I could find, especially the *Sphere*. My goal is offer such a backdrop by giving a birds-eye view of the marginalia found in hundreds of copies and dozens of editions of Sacrobosco's *Sphere*.[7] Following the example of William Sherman in tracing mostly anonymous readers, pursuing Robert Darnton's history of books 'from below',

---

[4] Thorndike (ed.), *The Sphere of Sacrobosco*, 112. 'quoniam prave est habitationis, sub climate non computatur'.

[5] Paradigmatic studies of reading often consider only individual readers: Lisa Jardine and Anthony Grafton, 'Studied for Action': How Gabriel Harvey Read His Livy', *Past and Present*, 129 (1990), 30–78; Ann Blair, *The Theater of Nature: Jean Bodin and Renaissance Science* (Princeton, 1997); William H. Sherman, *John Dee: The Politics of Reading and Writing in the English Renaissance* (Amherst, 1997); Nicholas Popper, *Walter Ralegh's 'History of the World' and the Historical Culture of the Late Renaissance* (Chicago, 2012). The wonderful study of astronomical marginalia by Owen Gingerich, *An Annotated Census of Copernicus' De Revolutionibus (Nuremberg, 1542 and Basel, 1566)* (Leiden, 2002), has set a high bar for studying marginalia; readers of *De revolutionibus* can hardly be identified as a kind of 'everyman'. William H. Sherman has put it, 'generalizations about Renaissance marginalia are hard to come by…. But there is a pressing need for information that will generate some larger pattersn across a wider range of books and readers': 'What Did Renaissance Readers Write in Their Books?' in *Books and Readers in Early Modern England: Material Studies*, ed. Jennifer Andersen and Elizabeth Sauer (Philadelphia, 2001), 119–37.

[6] Mordechai Feingold, *The Mathematician's Apprenticeship: Science, Universities and Society in England, 1560-1640* (Cambridge, 1984), 16–22; William H. Sherman, *Used Books: Marking Readers in Renaissance England* (University Park, PA, 2009). On the importance of textbooks and universities for understanding science during this period, see Patricia Reif, 'The Textbook Tradition in Natural Philosophy, 1600-1650', *Journal of the History of Ideas* 30 (1969), 17–32. On textbooks in this period, see Emidio Campi, Simone De Angelis, and Anja-Silvia Goeing (eds.), *Scholarly Knowledge: Textbooks in Early Modern Europe* (Geneva, 2008).

[7] At least 20 percent, I estimate, of the copies I have seen bear significant marginalia, a figure comparable with Sherman's study of several thousand books across multiple genres (cit. note 6). The tendency of rare books buyers until recently to value 'clean' exemplars suggests that more exemplars were annotated in the sixteenth century than we now have.

this article is primarily an effort to map the ways that the 'everyman' reader of Sacrobosco read the book.[8]

Before turning to annotations, however, in the first part of the article it is necessary to survey the contents of the *Sphere*, its uses, and how it transformed during the period as a site for innovation in commentary and visual presentation. Then, in the second part, I situate Sacrobosco within the university classroom by considering two readers we know something about, Beatus Rhenanus at Paris and Henricus Glareanus at Cologne. The majority of readers, of course, we know very little about, so in the third part I consider Beatus and Glareanus' annotations in comparison with my broader collection of largely anonymous annotations. In this third part, I offer a preliminary sketch[9] for a history of Sacrobosco's renaissance readership—a readership chiefly of university students and masters, given the *Sphere*'s longstanding place in the university *cursus*. In contrast to studies of reading that focus on individuals, I have tried to give a baseline for how this textbook was used; necessarily, my results are somewhat impressionistic (though perhaps not so impressionistic as focusing on one reader). Yet annotations witness to the possibly glacial and certainly profound ways that textbooks change a discipline, at a time when print catalyzed innovation in such textbooks. For heuristic reasons, therefore, I have organized these marginalia into three categories. First, I begin with Sacrobosco as most students encountered him, as an entry to basic astronomical terms, examining how readers used various marking strategies to integrate Sacrobosco into early modern letters, defined broadly as philosophical, historical, and literary education. Second, I consider marginalia that reflect the specific practice of comparing authorities, so powerfully deployed by Renaissance intellectuals. Third, I address the specifically mathematical literacy such marginalia reveal. Deploying the swelling commentaries and visual apparatus of new print editions, readers of Sacrobosco more firmly embedded the *Sphere*'s cosmology into renaissance learning even as they linked the *Sphere* with newer disciplines such as cosmography. In the process, I suggest, such apparatus discouraged high-level mathematical creativity, but also encouraged some readers to attend more to calculation.

---

[8] Sherman, *Used Books*; Robert Darnton, '"What Is the History of Books?" Revisited', *Modern Intellectual History* 4 (2007), 496. Ann Blair specificially outlines 'collective reading practices' as an emerging research direction for the history of reading science in 'Scientific Readers: An Early Modernist's Perspective', *Isis* 95 (2004), 420–30. See also, to a lesser degree, Blair's study of reference works and reading in *Too Much to Know: Managing Scholarly Information before the Modern Age* (New Haven, 2010). Her more recent approach complements the *histoire totale* of single readers found in the studies in footnote 5.

[9] Since there remain countless small textbooks in every collection of early books, most unexamined, this article is necessarily a 'preliminary sketch' that must be filled in and corrected.

The Expansion of Astronomical Textbooks

If any work counts as a textbook, it is Sacrobosco's *Sphere*, the longest-lived mathematical set text of Western European universities other than Euclid's *Elements*.[10] Yet the *Sphere* was by no means a stable object, and especially in print it often served as the skeleton on which to hang many different clothes. While the first printed editions of Sacrobosco's *Sphere* (1472) amounted to a few gatherings bare of images or commentary, by the 1570s the talented Jesuit mathematician Christoph Clavius used the *Sphere* as the frame on which to hang 500 pages of the most sophisticated astronomy of the period.[11] So far, historians have roughly divided the book's printed history into three phases. In the first, the book evolved a growing apparatus of images and commentary.[12] The second phase begins in the 1530s, when quarto and folio editions especially published in Venice and Paris gave way to small, inexpensive octavos published in Wittenberg, usually with Philip Melanchthon's oration on astronomy as preface.[13] In this second phase, Sacrobosco also became the model for a growing genre of textbooks, often under the name 'cosmography'.[14] In the final phase, in the second half of the sixteenth century, we find long, thick new textbooks roughly based on the *Sphere*, such as that by Clavius or the *Epitome astronomiae* of Michael Maestlin (Heidelberg,

---

[10] Anthony Grafton has pointed out the perplexing diversity of books that count as textbooks—namely, books read for and in class—during this period, in 'Textbooks and the Disciplines', in *Scholarly Knowledge*, 11-16. It appears that Boethius' works on arithmetic and music were replaced earlier than Sacrobosco: Ann E. Moyer, "The Quadrivium and the Decline of Boethian Influence," in Noel H. Kaylor and Philip E. Phillips (eds.) *A Companion to Boethius in the Middle Ages* (Leiden, 2012), 479–517.

[11] Christoph Clavius, *In Sphaeram Ioannis de Sacro Bosco commentarius* (Rome, 1570). Later editions acquired over a hundred pages of further material. See also Peter Barker, 'The Reality of Peurbach's Orbs: Cosmological Continuity in Fifteenth and Sixteenth Century Astronomy', in Patrick J. Boner (ed.), *Change and Continuity in Early Modern Cosmology*, Archimedes 27 (Berlin/New York, 2011), 7–32. An in-depth reading of Clavius' astronomy is found in James Lattis, *Between Copernicus and Galileo: Christopher Clavius* (Chicago, 1994).

[12] The visual reorganization of the *Sphere* is addressed by Owen Gingerich, 'Sacrobosco Illustrated', in *Between Demonstration and Imagination: Essays in the History of Science and Philosophy Presented to John D. North*, ed. Lodi Nauta and Arie Johan Vanderjagt (Leiden, 1999), 211–24; Isabelle Pantin, 'L'illustration des livres d'astronomie à la renaissance: l'évolution d'une discipline à travers ses images', in Fabrizio Meroi and Claudio Pogliano (eds.), *Immagini per conoscere: Dal Rinascimento alla Rivoluzione scientifica* (Firenze, 2001), 3–42; Jürgen Hamel, 'Johannes de Sacroboscos Sphaera', in Stephan Füssel (ed.), *Gutenberg-Jahrbuch*, 81 (2006), 113–36. On the visual strategies of Sacrobosco primers, see especially Kathleen M. Crowther and Peter Barker, 'Training the Intelligent Eye: Understanding Illustrations in Early Modern Astronomy Texts', *Isis* 104 (2013), 429–70. An analogous argument can be found in Steven Vanden Broecke, 'The Use of Visual Media in Renaissance Cosmography: The Cosmography of Peter Apian and Gemma Frisius', *Paedagogica Historica* 36 (2000), 130–50.

[13] F. R. Johnson, 'Astronomical Textbooks in the Sixteenth Century', in E. Ashworth Underwood (ed.) *Science, Medicine, and History: Essays on the Evolution of Scientific Thought and Medical Practice Written in Honour of Charles Singer*, vol. 1 (Oxford, 1953), 285–302. On the change from larger to smaller formats, see Owen Gingerich, 'Sacrobosco as a Textbook', *Journal for the History of Astronomy* 19 (1988), 269–73. The preface of the Wittenberg editions is analyzed by Isabelle Pantin, 'La lettre de Melanchthon à S. Grynaeus: avatars d'une défense de l'astrologie', in *Divination et controverse religieuse en Franx au XVIe siècle* (Paris, 1987), 85–101.

[14] Johnson, 'Astronomical Textbooks', 299-300.

1582).[15] In this section, I try to analyze the rapid expansion of the *Sphere* during the earlier part of its printed life, and consider what this implies about how readers used the text.

The early decades of Sacrobosco in print illustrates well the transition from manuscript to print: in uncertain fits and starts. For example, while most manuscripts include at least some images, the very first printed editions of Sacrobosco left only spaces for figures, perhaps intending readers to fill them in.[16] It may be these earliest printers followed manuscript copying practices, where the reader might commission or purchase a copy and later add rubrics or figures. How manuscript collections of astronomical works were organized also helps explain the growing collections of texts that surrounded Sacrobosco between 1472 and 1531. In manuscript, the *Sphere* of Sacrobosco is normally found in collections of astronomical texts. A copyist would, whether for himself or for another, include several similar items of interest in the same codex, most often grouping them together by subject.

From these manuscripts the historian can infer the uses to which such collections were put. Usually along with the *Sphere*, which provided the first rudiments of Ptolemaic two-sphere astronomy, one would usually find the *Theoricae planetarum* (sometimes attributed to Gerard of Cremona), the more advanced textbook on the planetary motions. One might find also an introduction to calculation and geometrical operations, and some applications such as a handbook on casting a horoscope in order to make judicial or medical astrological predictions. Or—perhaps most common of all—the manuscript would be rounded off with a *compotus* manual, used to determine the date of Easter.[17] Another especially important item was the *Alphonsine Tables*, used to develop astronomical medieval *calendaria* as a kind of almanach for predicting planetary positions. Ptolemaic astronomy as practiced from antiquity to the Renaissance was useful for determining the longitudes of the sun and other planets, and so for measuring the periods of stars and planets. These tasks required several texts. The *Theoricae planetarum* outlined the basic conceptual issues at play in this calculation. Roughly, the problem was that the sun (like other planets)

[15] Johnson, 'Astronomical Textbooks', 300-301.
[16] E.g. *Tractatum de spera* (Venice, [1472]), CUL Inc.4.B.3.8. Thanks to Roger Gaskell for this example.
[17] Examples of this sort that I have seen include: Houghton Library MS Typ 43; Ambrosiana A 183 Inf.; Ambrosiana E 12 Sup.; Ambrosiana H 75 Sup.; Ambrosiana I 90 Sup.; Ambrosiana M 28 Sup.; Ambrosiana M 35 Sup.; Ambrosiana N 50 Sup.; Ambrosiana T 69 Sup. For descriptions of many similar MSS, see Lynn Thorndike and Pearl Kibre, eds., *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin* (Cambridge, MA, 1937). A longer examination of one such MS (BL MS Arundel 66) is Hilary M. Carey, 'Henry VII's Book of Astrology and the Tudor Renaissance', *Renaissance Quarterly* 65 (2012), 661–710. A rare account of how fifteenth-century students may have used such manuscripts is James S. Byrne, 'The Stars, the Moon and the Shadowed Earth', PhD dissertation (Princeton, 2007), chap. 2.

does not rotate around the earth at a uniform rate. To account for this nonuniform movement, the *Theoricae* followed Ptolemy's *Almagest* by modeling the sun's movement on an eccentric deferent, a circle whose center is not the earth. Thus the true angular motion of the sun had to be calculated by using a trigonometric function that allowed for the difference between the radius of the deferent and the radius of the zodiac (centered on earth). In the case of the other planets, whose movement is even more irregular from the perspective of earth, Ptolemy and the *Theoricae* employed further geometrical constructions: the epicycle and the equant point. But the *Theoricae* gave a purely qualitative description of the arrangement of the Ptolemaic orbs, and thus was insufficient to actually calculate the locations of these planets. To find those, an astronomer needed astronomical tables (*ephimerides*) along with *canons*, or instructions for using those tables. It is not uncommon to find manuscript codices of astronomical works such as the *Sphere* or the *Theoricae* together with *Canones* and *Tabulae*, forming a complete set of astronomer's tools.

Printed editions of the *Sphere* did not immediately come with these practical components, though they were usually printed in collections. For their first two decades in print, most incunabula astronomy textbooks included three works: John of Sacrobosco's *Sphere*; Regiomontanus' *Contra Cremonensem*, which attacked the old thirteenth-century *Theoricae*; and his teacher Georg Peurbach's *Theoricae novae planetarum*, written to correct and replace the older book of the same title. None included the ephemerides necessary, however, to make calculations or to construct nativities.[18] Of course, one could purchase these separately; but their absence from the first *printed* collections including the *Sphere* suggests that these collections were primarily meant to impart a conceptual understanding of the heavens, not an operative one.[19]

Beginning in 1495 astronomical textbooks expanded rapidly, as did textbooks of the arts course in

[18] To see other works published with incunabula editions of Sacrobosco's *Sphere*, see Hamel, 'Johannes de Sacroboscos Sphaera'.

[19] These astronomical textbooks define, as much as instruments, the 'context of use' for astronomical practice. Their study will enrich debates over whether the 'utility' of astronomical models was primarily conceptual or operative. The poles of current scholarship on this question are defined by Jim Bennett, 'Knowing and Doing in the Sixteenth Century: What Were Instruments For?', *The British Journal for the History of Science* 36 (2003), 129–50; Adam Mosley, 'Objects of Knowledge: Mathematics and Models in Sixteenth-Century Cosmology and Astronomy', in Sachiko Kusukawa and Ian Maclean (eds.) *Transmitting Knowledge: Words, Images, and Instruments in Early Modern Europe* (Oxford, 2006).

general.[20] For one thing, textbooks began to include commentary. The first of two such commentaries was published at Paris in 1495 by the arts master Jacques Lefèvre d'Étaples, and until the 1530s was perhaps the most frequently reprinted commentary in Paris and Venice. In 1498, a colleague Pedro Ciruelo produced a similar commentary, based on a set of questions by Pierre d'Ailly a century earlier. Contemporary bindings show that sixteenth-century owners often bought multiple commentaries and had them bound together in *Sammelbände*, reading Lefèvre's and Ciruelo's commentaries in parallel.[21] Printers quickly sensed a market for multiple commentaries in one volume. In 1499, the Venetian printer Simon Bevilaqua published a compendium of commentaries on the *Sphere*, including Lefèvre's alongside other popular commentaries by Cecco Esculano, Francesco Capuano, as well as Peurbach's *Theoricae* and its own commentary by Capuano. From 1499 to the late sixteenth century, this genre of compendia on the *Sphere* swelled, reaching a high point in 1531 when Lucantonio Giunti published an edition in Venice that included no fewer than sixteen distinct texts, including the *Sphere*, old and new *Theorics*, medieval and Renaissance commentaries on them, and orations in praise of the quadrivium.

As Sacrobosco's *Sphere* evolved, the genre of the *Sphere* diversified and became more technically demanding.[22] Two changes command special attention. First, the *Sphere* diversified into a broader genre that took on elements from Ptolemy's newly available *Cosmography*, so merging with one of the most distinctive disciplines of Renaissance practical mathematics. At the same time, second, new versions and indeed commentaries on Sacrobosco's *Sphere* began to mention, and sometimes explain, newer technical questions in mapping and

[20] On the move toward textbooks generally in the sixteenth century, see Charles B. Schmitt, 'The Rise of the Philosophical Textbook', in Charles B. Schmitt et al. (eds.) *The Cambridge History of Renaissance Philosophy* (Cambridge, 1988), 792–804; Campi, Angelis, and Goeing, *Scholarly Knowledge*. Isabelle Pantin has also identified 1495 as the beginning of a new phase of printed commentaries in the *Theorica* genre: Pantin, 'L'illustration des livres d'astronomie à la renaissance: l'évolution d'une discipline à travers ses images', in *Immagini per conoscere: Dal Rinascimento alla Rivoluzione scientifica*, ed. Fabrizio Meroi and Claudio Pogliano (Firenze, 2001), 8ff.

[21] For example, Houghton gen typ 515.16.764, a volume that binds together Lefèvre's *Textus de Sphere* (Paris, 1516) with Ciruelo's commentary *Uberrimum sphere* (Paris, 1498); Newberry (Vault) Ayer 6 .S2 1507 binds together Lefèvre's *Textus* (Paris, 1507) with Ciruelo (Paris, 1515). This combination is also found in the library of Beatus Rhenanus as BHS [Bibliothèque humaniste de Sélestat] K950b.

[22] This diversification holds true more generally for mathematics in the period; see Brigitte Hoppe, 'Die Vernetzung der Mathematisch ausgerichteten Anwendungsgebiete mit den Fächern des Quadriviums in der Frühen Neuzeit', Irmgarde Hantsche (ed.) in *Der "mathematicus" zur Entwicklung und Bedeutung einer neuen Berufsgruppe in der Zeit Gerhard Mercators* (Bochum, 1996), 1–33.

calculation.[23]

First, the genre of the introductions to spherical astronomy diversified. Even before print, there had been a number of introductions that merited the title 'sphere'; Sacrobosco's work became the most popular introduction, and so defined the genre.[24] But in the early sixteenth century, new introductions to the *Sphere* began to incorporate map-making or cosmography as one of the main aims of the genre. Such works followed the example of Ptolemy's *Geography*—or, *Cosmography*, as it was more commonly translated at first—which had been newly discovered in the early fifteenth century.[25] Notably, Ptolemy's approach to map-making offered mathematical and astronomical techniques for projecting a cartographic grid—it determined the longitudes and latitudes of places on earth from their positions relative to the night sky. That is, Ptolemy focused on the mathematical link between the cosmic sphere and the earthly sphere. In this vein, new introductions to the sphere included the *Cosmographiae introductio* of Martin Waldseemüller (1507), and Peter Apian's *Cosmographicus liber* (1524).[26] For example, in Paris Jean Fernel and Oronce Fine each produced comparable textbooks through the presses of Simon Colines, even as Fine helped Colines publish the *Sphere* commentary of Lefèvre d'Étaples.[27] Until 1528, when he gave up his affair with mathematics to become a respectable physician, Fernel was absorbed in the world of mathematical instruments, and

[23] In the second half of the sixteenth century, this genre dropped some of these canonical texts, incorporating more sixteenth-century authors and growing increasingly technical. Notable examples are Elia Vinetus, *Sphaera emendata* (Paris, 1572) (which included commentaries by Gemma Frisius) and Clavius, *In Sphaeram Ioannis de Sacro Bosco commentarius*. But collectors of mathematical books such as the French lawyer Jean I du Temps, active at the beginning of the seventeenth century, still chose the edition of Giunta published in 1531 to represent the tradition of Sacrobosco's Sphere, though he supplemented it with the works of Oronce Fine on cosmography and instruments. See Alexander Marr, 'A Renaissance Library Rediscovered: The "Repertorium Librorum Mathematica" of Jean I du Temps', *The Library* 9 (2008), 428–70, entries 10 and 7, respectively (at pages 442–4). Marr argues that du Temps' library is more representative of mathematical libraries than, for example, John Dee's or Bernardino Baldi's.

[24] Rival textbooks were written already in the thirteenth century by Grosseteste, Peckham, and Campanus. Thorndike, *The Sphere of Sacrobosco*, 23-28.

[25] On the title *Cosmographia*, see James Hankins, 'Ptolemy's Geography in the Renaissance', in *Humanism and Platonism in the Italian Renaissance*, 2 vols., Storia e Letteratura: Raccolta di Studi e Testi 215 (Rome, 2003), vol. 1, 457–68. More generally, see also Zur Shalev and Charles Burnett, eds., *Ptolemy's Geography in the Renaissance*, Warburg Institute Colloquia 17 (London, 2011).

[26] Larger cosmographies such as Sebastian Münster's *Cosmographia* (1544) bridge this genre with classical and medieval encyclopedic traditions. For an account of these traditions, see Matthew McLean, *The Cosmographia of Sebastian Münster: Describing the World in the Reformation* (Aldershot, 2007), 47–105. On the character of cosmography in this period, see Adam Mosley, 'The Cosmographer's Role in the Sixteenth Century: A Preliminary Study', *Archives internationales d'histoire des sciences* 59 (2009), 423–439.

[27] Fine designed the titlepage for Simon Colines' new edition of Lefèvre's *Textus de Sphaera* when Colines took over the press from Henri Estienne in 1521. The press continued to produce the book, with newly added marginalia, to 1538.

in his *Monalosphaerium* (1526) and *Cosmotheoria* (1528) he reworked the subject of the *Sphere* with much greater attention to the geometry and instruments needed to map the heavens.[28] Similarly, Oronce Fine successfully bid for a position as *lecteur royal* in Francis I's new Collège Royal with his compendium *Protomathesis* (1532), of which the third part was a *Cosmographia*—Colines later republished it separately as *De mundi sphaera, sive cosmographia* (1542). Introductions to the *Sphere* often merged with the genre of geography, resulting in a new range of astronomical texts.

The second major change in astronomical textbooks was that, even as the genre of the sphere diversified, commentaries also became much more technically sophisticated than Sacrobosco's text itself. Like Sacrobosco, all the treatises began with definitions of points and spheres from geometry, then moved to the structure of the heavens, and finally described relationships between heavens and the habitable parts of the earth. Unlike Sacrobosco, new authors transgressed the old theoretical bounds of the *Sphere*, entering into practical concerns. They were interested in instruments and calculation, in applying mathematics to the world, often with an eye to the art of mapping. While I know of one medieval commentator who mentioned an instrument in the context of Sacrobosco's *Sphere*,[29] the majority simply tried to transmit a qualitative understanding of the cosmos to students, without meaning to make the student a practitioner. In the early sixteenth century, textbooks increasingly aimed to teach students how to manipulate the tools of the astronomer. By 1490, some Venice editions were including a short introduction to geometrical terms needed for doing astronomy: curves, right and acut angles, sections of circles, and parts of spheres such as poles.[30] In Paris, Lefèvre's *Textus de Sphaera* (1495) framed Sacrobosco within a short primer on sexagesimal arithmetic, tables for calculating the rising and setting time of the sun and planets, and accounts of the locations of cities.[31] These new elements may be related to new treatises on spherical geometry available from Greek antiquity, such as pseudo-Proclus.[32] Humanists frequently mentioned such newer texts in their commentaries on

---

[28] Sir William Sherrington, *The Endeavour of Jean Fernel* (London, 1946), 18.

[29] Robert of England described the 'invention of a clock' in his eleventh lecture on Sacrobosco's *Sphere.* Thorndike, *The Sphere of Sacrobosco*, 180–181 (translated at 230–1).

[30] Sacrobosco, *Sphaera mundi* (Venice, 1490), a4r-a5r.

[31] Lefèvre, *Textus de sphera* (Paris, 1495), a3r.

[32] In fact, this short treatises was chapter 4, 5, 15, and 3 (in that order) of the *Elementa astronomiae* of Geminus. Despite a partial Latin translation by Giorgio Valla from 1490, it was mostly read in the full Latin translation by Thomas Linacre (Venice, 1499). It is discussed in Georg Joachim Rheticus' Wittenberg lecture notes (1530s) so it early was added to classrooms. For more bibliography, see Robert B. Todd, 'Geminus and the Ps.-Proclan *Sphaera*, in Virginia Brown, ed., *Catalogus translationum et commentationum* (Washington, DC: The Catholic

Sacrobosco, as Melanchthon did in his introduction to the Wittemberg edition, and the editor (probably Oronce Fine) who added the marginal notes to editions of Lefèvre's *Textus de Sphaera* from 1521 onward, supplying Greek references and *renvois* to the technical literature of antiquity.

These printed astronomical texts, especially when seen against the backdrop of medieval manuscript collections of astronomical writings, raise further questions of use. The more sophisticated introductions to Ptolemy's masterwork, the *Almagest*, found a strictly limited audience. Regiomontanus' *Epytoma in Almagestum Ptolemei* (Venice, 1496), famously the primary source of Nicolaus Copernicus' knowledge about Ptolemy, was never reprinted.[33] Early on, therefore, the kinds of resources a reader needed to calculate the locations of stars, to compare maps, and to test authorities, were not widely available. Meanwhile, it would have been quite difficult to apply the astronomical knowledge presented in the simpler *Sphere* and *Theorica*, for they included no tables for actually transferring knowledge about the heavenly bodies from the page to prediction. As Lefèvre noted in his own treatment of the *Theoricae* (Paris, 1503), one wishing to actually ascertain or predict the positions of the bodies should go elsewhere—to Ptolemy himself.[34] In Lefèvre's view, the limitations of such textbooks to mostly theoretical inquiry was not a particular problem. While he did not forbid the reader from astrology, Lefèvre tinged his description of such practical astronomy with prejudice against the prognostications of the heathen, who understood but misapplied the knowledge of the heavens. His student-cum-colleague Josse Clichtove, who faithfully commented on Lefèvre's short treatise in exhaustive (and sometimes exhausting) detail, reiterated the point in 1517. The astronomy of Sacrobosco and the *Theorica* tradition, he told readers, should be for contemplation and not for prognostication.[35] This pedagogical ambivalence toward astronomy raises questions for historians. For what ends did early modern astronomy students turn to Sacrobosco? Did they simply turn to the night sky, in awe of the first mover as they used their imaginations to apply the circles of their pages to the spheres in the skies? Or was this awe meant to be inculcated only in the pages of books, within the shadows of college yards and libraries?

University of America Press, 2003), 8:12–5.

[33] On Copernicus' reading, see bibliography in Robert S. Westman, *The Copernican Question: Prognostication, Skepticism, and Celestial Order* (Berkeley, 2011), chap. 2. Westman also eloquently displays the value of studying the *whole* range of astronomy textbooks, not simply the most innovative or advanced works.

[34] Eugene F. Rice Jr (ed.), *The Prefatory Epistles of Jacques Lefèvre d'Étaples and Related Texts* (New York, 1972), 112–114.

[35] Ibid., 391–395.

The *Sphere* in the Classroom: Beatus Rhenanus and Henricus Glareanus

Let us enter the Renaissance classroom through the experiences of the Alsatian humanist Beatus Rhenanus (1485-1547) and the Swiss polymath Henricus Glareanus (1488-1563). Both were exceptional readers in the sense that they ended their lives as well-published authors and internationally-regarded humanists, members of the close circle of humanists Erasmus cultivated along the Rhine after his arrival in Basel in 1514.[36] But from the perspective of the history of science, they can serve as a kind of 'everyman' insofar as they embodied the ideal of encyclopedic erudition that became the ideal of education in early modern Europe.[37] While diligent students, in other words, they can hardly be called astronomers, even though Glareanus was known for skill in other mathematical disciplines such as cosmography and music theory.

Beatus Rhenanus came to Paris in the summer of 1503 to study with Lefèvre d'Étaples at the Collège du Cardinal Lemoine. Since the early 1490s, Lefèvre had been renovating the range of university textbooks, including new introductions to and commentaries on the standard mathematical works of the quadrivium as well as the philosophical works of Aristotle: logic, natural philosophy, followed in later years by moral philosophy and metaphysics. By the time Beatus came to Paris, a student at Cardinal Lemoine could study the whole arts course using nothing but the exciting new 'cursus Fabri'. On arrival in 1503, Beatus bought many books, but the only ones closely annotated with study and lecture notes are the editions produced by Lefèvre and his other teachers.[38] Given

---

[36] On Beatus as humanist, see François Heim and James Hirstein (eds.) *Beatus Rhenanus (1485-1547): lecteur et éditeur des textes anciens* (Turnhout, 2000). On Glareanus, see the recent studies assembled by Iain Fenlon and Inga Mai Groote (eds.) *Heinrich Glarean's Books: The Intellectual World of a Sixteenth-Century Musical Humanist* (Cambridge, 2013).

[37] As David Lines has argued, it is unhelpful to contrast 'universities' (or 'scholastic') with 'humanist': David A. Lines, 'Humanism and the Italian Universities', in *Humanism and Creativity in the Renaissance: Essays in Honor of Ronald G. Witt*, ed. Christopher S. Celenza and Kenneth Gouwens (Leiden, 2006), 327–46. Glareanus too was a humanist who spent most of his life teaching in universities.

[38] Beatus' books bought during his grammar school and universities studies are listed in Gustav Knod, *Aus der Bibliothek des Beatus Rhenanus: ein Beitrag zur Geschichte der Humanismus* (Leipzig, 1889). A comparison of their annotations may be found in Richard J. Oosterhoff, 'Mathematical Culture in Renaissance Paris: University, Print, and the Circle of Lefèvre d'Étaples' (PhD Dissertation, University of Notre Dame, 2013), Appendix C, 410-412. Foundational studies of Beatus' student notes are Emmanuel Faye, 'Beatus Rhenanus lecteur et étudiant de Charles de Bovelles', *Annuaire des Amis de la Bibliothèque Humanist de Sélestat*, 1995, 119–38; Emmanuel Faye and Michel Ancey, 'Le cours de métaphysique de 1504 pris en note par Beatus Rhenanus au Collège du Cardinal Lemoine. Édition et traduction des propositions 1 à 3', *Annuaire des Amis de la Bibliothèque Humanist de Sélestat*, 1995, 139–42; Emmanuel Faye, 'Nicolas de Cues et Charles de Bovelles

the low interest in mathematics in fifteenth-century University of Paris, it is striking that Beatus' first year at Paris included three works on mathematics, alongside include Lefèvre's edition and commentary on the texts of Porphyry and Aristotle's logical *Organon*, and a second volume of Lefèvre's paraphrase of Aristotle's various works on natural philosophy. In his prefaces, Lefèvre argued that mathematics should be restored to its old place in the University of Paris, implying that study of Sacrobosco and geometry had become a dead letter of the university statutes. At Paris, it appears that the distinctive academic culture of each residential college determined what and how students actually read. The *cursus* for the arts course leading to the bachelor degree was not set by university statute.[39] Even the requirements for incepting MAs stated only the vague requirement that the candidates have heard 'at least one hundred lectures on mathematical topics'.[40] (An official recorded the usual interpretation: 'this is interpreted by the Faculty [of Arts] thus: that it is enough to have heard one book of mathematics, such as the treatise on the sphere, and to be in the process of hearing another book with the intention of hearing it until the end, without lying'. [41]) Mathematics indeed kept a low profile in the statutes of the medieval University of Paris. Beatus' purchases suggest that, at least at the Collège du Cardinal Lemoine, the story was different. In Lefèvre's own classroom, mathematics were of primary importance, studied early in the BA—primary, in the senses of being basic, but also fundamental to later disciplines.

As a student, Beatus read an edition with Lefèvre's commentary published in Paris in 1500, though he later

dans le manuscrit 'Exigua pluvia' de Beatus Rhenanus', *Archives d'histoire doctrinale et littéraire du moyen âge*, 1998, 415–50.

[39] In fact, would-be bachelors were not even inscribed in the university rolls until they graduated, and were solely registered in their respective colleges. See, for example, the register edited by James K. Farge, ed., *Students and Teachers at the University of Paris: The Generation of 1500. A Critical Edition of Bibliothèque de l'Université de Paris (Sorbonne), Archives, Registres 89 and 90* (Leiden, 2006). The actual practice at Paris is probably mirrored in the more detailed statutes of universities in Northern Europe that were founded by Parisian masters in the fourteenth and fifteenth centuries, such as the universities of Cologne and Vienna (for an example, see discussion below of Cologne). See Michael H. Shank, *"Unless You Believe, You Shall Not Understand": Logic, University, and Society in Late Vienna* (Princeton, 1988), 35–36. Compare the statutes also at Erfurt: H. Wissenborn (ed.), *Acten der Erfurter Universität*, vol. 2 (Halle, 1881), 2:134. For a fuller discussion of the requirements at Paris, see Richard J. Oosterhoff, 'Mathematical Culture in Renaissance Paris', 56-61.

[40] H. Denifle and E. Châtelaine, eds., *Chartularium universitatis Parisiensis* (Paris, 1889), 2:678. 'Item, quod audivistis centum lectiones de Mathematica ad minus. (*Istud per facultatem sic est interpretatum quod sufficit audivisse unum librum totalem mathematice, sicut tractatum de Spera, et alium librum actu audire cum spe audiendi usque ad finem sine fraude*)'.

[41] Book of the Chancellor: fol. 5r, 6r-v, cit. Charles Thurot, *De l'organisation de l'enseignement dans l'Université de Paris au Moyen-Âge* (Deis, 1850), 51: 'Istud per facultatem sic est interpretatum, quod sufficit audivisse unum librum mathematicae, sicut tractatum de sphera, et alium librum actu audire cum spe audiendi usque ad finem sine fraude'.

purchased two more copies. In 1504 he acquired an edition with commentary by Pedro Ciruelo and the older *quaestiones* of Pierre d'Ailly. In 1505, he bought one of the Venice editions that included several further commentaries, perhaps to supplement his own lecturing as an incepting Master of Arts. A fastidious note-taker and a compulsive book-buyer, Beatus put his *ex libris* on the title pages of all his purchases, or only the first title page, when he bought several books in a single *Sammelband*. This is the case with Beatus' Sacrobosco: it is bound at the end of a volume of three works, following some compendious introductions to practical and theoretical arithmetic and geometry by Lefèvre, Josse Clichtove, and Charles de Bovelles, and then Lefèvre's advanced editions of number and music theory.[42] Beatus therefore likely read his Sacrobosco in the context of his study of natural philosophy and logic, and probably after having studied some elementary arithmetic and geometry (the more advanced treatises on number theory and music theory are mostly blank).

Henricus Glareanus most likely read and annotated his Sacrobosco while in Cologne, a northern center of education and textbook printing with prestige second only to Paris.[43] A native of the Swiss town Glarus, Glareanus arrived at the University of Cologne in 1507 after attending the grammar school at Rottweil in Württemburg, earning the BA in 1509, and the MA in 1510. As at Paris, the masters directed studies as well as living arrangements in each residential college or *bursa*; Glareanus first studied at the *Bursa Montis* and then taught there as a master. For bachelors, university statutes at Cologne were little clearer than those at Paris, and mentioned nothing of mathematics. The statutes specified mathematical requirements for masters more carefully: bachelors who hoped to incept as masters should have spent two months in lectures on the *Sphere*, as well as ten weeks on parts of Euclid, the *Theorica*, and perhaps some of John Pecham's textbook on optics, the *Perspectiva communis*.[44] Perhaps the most

[42] The *Sammelband* is BHS K 1046, including: (a) Lefèvre et al., *Epitome in libros arithmeticos* (Paris, 1503), (b) Jordanus Nemorarius and Lefèvre, *Elementa arithmetica, musicae* (Paris, 1496), (c) Sacrobosco, *Textus de sphera* (Paris, 1500). Hereafter, 'Beatus' Sacrobosco'.

[43] The recent study of Glareanus' library and annotations begins with Iain Fenlon, 'Heinrich Glarean's Books', in *Music in the German Renaissance: Sources, Styles, and Contexts*, ed. John Kmetz (Cambridge, 1995), 74–102; Cristle Collins Judd, *Reading Renaissance Music Theory: Hearing with the Eyes* (Cambridge, 2000), 130–176. More recently, see Iain Fenlon and Inga Mai Groote (eds.) *Heinrich Glarean's Books*; Anthony Grafton and Urs Leu, *Henricus Glareanus's (1488-1563) Chronologia of the Ancient World: A Facsimile Edition of a Heavily Annotated Copy Held in Princeton University Library* (Leiden, 2013).

[44] The statutes of Cologne are in Franz J. von Bianco, *Die alte Universität Köln und die spätern Gelerhten-Schulen dieser Stadt, vol. 1/1: Die alte Universität Köln* (Cologne, 1856), 68. 'Item talis [Bacalarius temptandus] debet audivisse ultra illos in aliquibus Scolis publicis alicujus Universitatis in qua protunc fuerunt quinque Regentes magistri in artibus, libros infrascriptos: Physicorum ex toto; de celo et mundo; de generatione et corruptione; Metheororum; parva naturalia quo ad quatuor libros; de sensu et sensato; de sompno et vigilia, de memoria et

remarkable implication of these statutes is that students were not expected to have read Sacrobosco until they had already earned the BA.

Glareanus' Sacrobosco makes it plausible that he read the *Sphere* most intensively during his preparation for the MA. His working copy is a plain quarto edition printed in Paris in 1493.[45] It is bound with another copy of the *Sphere*, in the edition of Santritter (Venice, 1488), which includes the oration of Regiomontanus and Peuerbach's *Theorica*—Iain Fenlon and Inga Mai Groote report that this copy, however, has fewer annotations, mostly focused on the *Theorica*.[46] Glareanus did not annotate his working copy, however, until at least 1507; near the end of the book, he nuanced Sacrobosco's account of the torrid climes by reporting the experience of Amerigo Vespucci 'in the *Cosmographia*'.[47] The relevant letter of Vespucci had been published in 1507 with the *Cosmographiae introductio* that accompanied Martin Waldseemüller's map introducing the New World as 'America'.[48] Therefore Glareanus annotated this copy of the *Sphere* after April 23 1507, when the *Cosmographiae introductio* was published—that is, after he had already held the degree of bachelor.

Beatus and Glareanus' copies of Sacrobosco add to our growing picture of the renaissance classroom. As printed books became more and more common in classrooms, students sometime copied into the margin the detailed commentary of their teacher, keyed to particular words or arguments—in rare cases, we can compare several identical sets of student notes, providing us several views on the same classroom.[49] Though printed books were more

---

reminiscentia; de longitudine et brevitate vite; Spheram mundi; Theoricas planetarum; tres libros Euclidis; Perspectivam communem; aliquem tractatum de proportionibus, et aliquem de latitudinibus formarum; et aliquem in musica; et aliquem in aritmetica. Et sex libros Ethicorum et metaphisicam'. The statutes go on to specify how long most of these books should be 'read'.

45 Munich, Universitätsbibliothek, Inc.lat. 310#6. Hereafter 'Glareanus' Sacrobosco'. I have not examined the *Sammelband*, and I use the digital version available at <http://epub.ub.uni-muenchen.de/11720/>.

46 Iain Fenlon and Inga Mai Groote, 'Heinrich Glarean's Books', in Fenlon and Groote (eds.), *Heinrich Glarean's Books*, 322, nos. 60 and 61.

47 Glareanus' Sacrobosco, B1r. 'Velut Americus Vesputius qui usque ad circulum anarcticum fere passim duxit […] Vide latius Americum Vesputium in Cosmographia'.

48 *Cosmographiae introductio cum quisbusdam geometriae ac astronomiae principiis ad eam rem necessariis insuper quatuor Americi Vespucii navigationes universalis Chosmographiae descriptio* (St. Dié: [Vautrin Lud Nicolas Lud], 1507). Most likely the book itself was composed by Matthias Ringmann, not his friend Waldseemüller, though both contributed prefatory letters to the work: Franz Laubenberger, 'The Naming of America', trans. Steven Rowan, *The Sixteenth Century Journal* 13 (1982), 91–113.

49 For example, Jürgen Leonhardt has shown this for annotated *Sammelbände* from Leipzig; 'Classics as Textbooks: A Study of the Humanist Lectures on Cicero at the University of Leipzig, Ca. 1515', in Campi et al. (eds.), *Scholarly Knowledge*, 89-112. This practice should be distinguished from the equally important practice of copying important annotations from one copy of a book into another, especially famous for certain copies of

widely used in classrooms, the ancient practice of taking notes from dictation (the medieval term *reportatio* declined in use) continued throughout the early modern period.[50] Without multiple copies from the same classroom, it can be difficult to prove when this occurred, but large blocks of texts found in some books includes clues to their source in class discussions.[51] Beatus authenticated his schoolbooks with the phrase 'manu propria',[52] and then inserted large blocks of text into the margins. In the process, when the lecturer spoke too quickly or gave a name not yet familiar, Beatus sometimes left spaces that he apparently meant to fill in later.

[insert figure here]

Figure 1. Lefèvre, *Textus de sphera* (Paris, 1500), BHS K1046a, a1v, detail.

In this list of 'modern astronomers', the name 'Johannes de' precedes a blank, followed by the name 'Georgius Purbachius': Beatus had missed the name 'Regiomontanus' in the lecture.

Beatus' Sacrobosco reflects the experience of a university student fresh from the grammar school. Glarean's book instead was likely intended as a reference for students, who might copy his notes into their own copies of Sacrobosco. Recent studies of Glarean's later teaching notes suggest that when he taught in the *bursae* at Basel, Paris and Freiburg, he shared his library and expected students to read the notes that filled his books. Some of his notes resurface in students' books, copying the thickly penned capitals of headings and the smaller cursive of longer notes, and in some cases even copying the notes word for word.[53] Glareanus regularly addresses his notes to

Copernicus' *De revolutionibus* that circulated among Lutheran astronomers, including Tycho Brahe: Owen Gingerich and Robert S. Westman, 'The Wittich Connection: Conflict and Priority in Late Sixteenth-Century Cosmology', *Transactions of the American Philosophical Society* 78 (1988), i–148. The notes of Erasmus Reinhold and Jofrancus Offusius suggest a circle of disciples, but the work's difficulty makes it likely these were specialist readers, outside the university context; see Gingerich, *Annotated Census*, xix-xxi.

[50] See Ann Blair, 'Student Manuscripts and the Textbook', in Campi et al. (eds.), *Scholarly Knowledge*, 49; L. W. B Brockliss, *French Higher Education in the Seventeenth and Eighteenth Centuries: A Cultural History* (Oxford, 1987), 192; Françoise Waquet, *Parler comme un livre: l'oralité et le savoir, XVIe-XXe siècle* (Paris, 2003).

[51] Lefèvre, *Textus de sphaera* (Paris, 1500), BHS K1046c (i.e. Beatus' Sacrobosco); BnF res.v.209; Glogoviensis, *Introductorium compendiosum* (Cracow, 1513), in the private collection of Owen Gingerich.

[52] Jacques Lefèvre d'Étaples, Josse Clichtove, and Charles Bovelles, *Epitome compendiosaque introductio in libros arithmeticos* (Paris, 1503), BHS 1046a, titlepage. 'Est Beati Rhynavv Eschletstattini 1.4.0.3 Parrhisiis. Ma[nu] pro[pria]'.

[53] Inga Mai Groote and Bernhard Köble, 'Glarean the Professor and His Students' Books: Copied Lecture Notes', *Bibliotheque D' Humanisme et Renaissance: Travaux et Documents* 73, no. 1 (2011): 61–91; Inga Mai Groote, Bernhard Köble, and Susan Forscher Weiss, 'Evidence for Glarean's Music Lectures from His Students' Books: Congruent Annotation in the Epitome and the Dodekachordon', in Fenlon and Groote (eds.), *Heinrich Glarean's Books*, 209.

readers. Years later, in another similarly marked-up textbook, he explained the reason for his annotations: 'Glarean to the reader: What we have written here by our own hand, dear reader, you should ponder again and again, and it should be imprinted on your memory. You will find that this is very profitable for you, and it will serve you as a sort of open door to the rest of the material in this book'.[54] In his notes on Sacrobosco, Glareanus sometimes likewise addressed his reader directly. 'But the reader will remember that you call "uninhabitable" a zone which can hardly be lived in, so the poets affirmed the truth [when Virgil and Ovid delineated the torrid zone]'.[55] Perhaps as early as his years at Cologne, Glareanus intended that others use his own library.

We should imagine Beatus and Glareanus in a context of pedagogical experimentation. Students wrote their annotations in the margins of new printed textbooks. Sometimes, as Beatus' marginalia show, this was done in class—perhaps with the equivalent of a blackboard, so the master could share details. Glareanus' notes show that some lecture notes, however, could be copied by students out of their master's copy. This pedagogical space frames the context in which the growing population of university scholars read Sacrobosco.[56]

But what did studying astronomy look like, for most of these readers? Before considering this question in more detail, a caution is in order. Glareanus and Beatus had distinctive experiences, which only offer a limited picture of Sacrobosco's readership. Beatus' habits as an unusually punctilious reader served him well as editor of ancient works and a corrector in Amerbach's print shop.[57] At Paris, Basel, and Freiburg, Glareanus became a popular teacher, known for his sense of humour as well as his talent for clear exposition—evident in the notes he shared with students. Furthermore, they represent a distinctively northern education only in the early part of the century. By the late 1560s at Wittenberg, the *Sphere* was available in smaller octavos, and we have lecture notes that

[54] Cited and translated by Anthony T. Grafton and Urs B. Leu, 'Chronica est unica historiae lux: How Glarean Studied and Taught the Chronology of the Ancient World', in *Heinrich Glarean's Books*, 248-79, at 262. 'Glareanus Lectori: Quae nostra manu huc pinximus, studiose lector, etiam atque etiam tecum meditare ac tuae infigas memoriae, videbis magnum huius rei tibi fructum, ac velut tibi apertam, ad omnia in hoc libro sequentia, ianuam'.

[55] Glareanus' Sacrobosco, B1r. 'Sed meminerit Lector, inhabitabilem diceris zonam quae vix inhabitatus potest, hinc verum poetae dixerant'.

[56] The growth in number, size, and cultural importance of universities during the sixteenth century is surveyed by Jacques Verger, 'Patterns', in H. de Ridder-Symoens and Jacques Verger (eds.), *A History of the University in Europe, Vol. 1: Universities in the Middle Ages* (New York: Cambridge University Press, 2003), 35–68.

[57] Beatus' career as an exceptional corrector is outlined in Anthony T. Grafton, *The Culture of Correction in Renaissance Europe* (London, 2011), passim.

survive in notebooks, independently from textbooks.[58] And universities famous for medicine such as Bologna and Padua, where astronomy was important because of astrology's place in medicine, hired professors of mathematics in a way that did not happen in the north until Oronce Fine became royal professor of mathematics in 1531.[59] With teachers dedicated to mathematics, Italian universities could set schedules for these disciplines that were more or less standard across the university—rather than peculiar to individual colleges or *bursae*. Besides being limited as representations of a northern experience, Beatus and Glareanus' Sacrobosco are limited as part of a university readership. The marginalia in Gabriel Harvey's Sacrobosco, for example, reveal a kind of reading outside the university, aimed instead towards courtly utility—a kind of reading to be taken up at another time.[60] Given these limitations, therefore, to gain a broader perspective on how Sacrobosco was read in sixteenth-century Europe, the experience of Beatus and Glareanus should be read in dialogue with the many anonymous annotations in copies of Sacrobosco from the sixteenth century. On to the marginalia.

Kinds of Reading

We might wonder whether readers such as Beatus or the students of Glareanus learned how to read in a way distinctively astronomical or mathematical. Did Sacrobosco, in other words, foster what we might call 'mathematical literacy'? In order to see what is distinctively mathematical or astronomical in these marginalia, we must first consider the larger context of reading habits shared by most readers of the time. For heuristic purposes, I have found that student notes on the *Sphere* fall into three broad categories of reading, which I now discuss in turn: (1) the mining of astronomical knowledge for the sake of other literary texts; (2) critical comparison of authorities on astronomical knowledge; and (3) calculations. The most distinctively mathematical forms of reading appear chiefly in the latter two categories.

[58] Such manuscript notes are described by Owen Gingerich, 'From Copernicus to Kepler: Heliocentrism as Model and as Reality', *Proceedings of the American Philosophical Society* 117, no. 6 (1973): 516–519.

[59] The exception reveals the how unusual this was at Paris: Jean-Patrice Boudet, 'A "College of Astrology and Medicine"? Charles V, Gervais Chrétien, and the Scientific Manuscripts of Maître Gervais's College', *Studies in History and Philosophy of Science* 41, (2010), 99–108. On the early professorships of mathematics at Padua and Bologna, see Paul F. Grendler, *The Universities of the Italian Renaissance* (Baltimore, MD, 2002), 416–422.

[60] I plan to address Harvey's reading of Sacrobosco in another article.

1. For the Sake of Letters

No doubt many early modern students held the two-sphere astronomy of the *Sphere* in no more esteem than many secondary school students hold calculus today: a technical hoop, to be avoided if possible. Yet the rationale for reading Sacrobosco was evident. Sacrobosco was a useful component in the cycle of arts that formed functional citizens of the Republic of Letters—astronomy was to embed students more firmly in the language and cosmos of the learned. Beatus' annotations bring this motivation to the fore. He carefully underscores several lines in Lefèvre's letter prefacing his commentary on the *Sphere*. There Lefèvre recalled how his Greek tutor, the émigré George Hermonymus, convinced him to study mathematics. Beatus underlined a passage in which Hermonymus reported that 'mathematics (if we believe Plato in book 7 of the *Republic*) not only is useful for the republic of letters, but also has the greatest import for the civil republic. Thus (Plato thinks) those who have the best natures should especially be taught in mathematics'.[61] Beatus observed in the margin that it was George of Trebizond, the controversial Greek humanist, whom Lefèvre and Hermonymus held up as a good example of such training. As Beatus noted, the story shows 'the not insignificant usefulness of mathematics'.[62] Lefèvre and his circle of students and colleagues found in mathematics a means for the reform of university education—indeed, the republic of letters as well as civil rule.[63]

Such an education increasingly included technical subjects. Lefèvre's story, and Beatus' marginalia, hints at the increasing profile of mathematics in literary life over the course of the sixteenth century, as education became the point of entry into an expanding culture of civility. By the later sixteenth century, polite acquaintance with the geometry of artillery or navigation was hardly unusual in elite education. Such cultured appreciation of the arts depended on a tradition of books such as Sacrobosco's, books that integrated mathematics into a predominantly literary education.[64] Indeed, genteel education sometimes mingled mathematics and artisanal practice, for instance in

---

[61] Beatus' Sacrobosco, a1v. 'Mathememata inquit [Hermonymus] que (si Platoni septimo de republica credimus) non modo rei publice litterarie, sed et civili momentum habent maximum; et in his (ut sentit Plato) precipue erudiendi sunt qui naturis sunt optimis' (emph. Beatus). This letter is also published in Rice (ed.), *Prefatory Epistles*, ep. 8.

[62] Beatus' Sacrobosco, a1v. 'Mathematum non modica utilitas'.

[63] Some of these motivations came from Nicholas of Cusa, whose *theologia mathematica* became popular in Lefèvre's circle: Stephan Meier-Oeser, *Die Präsenz des Vergessenen: Zur Rezeption der Philosophie des Nicolaus Cusanus vom 15. bis zum 18. Jahrhundert*, Buchreihe der Cusanus-Gesellschaft 10 (Münster, 1989), 36–61.

[64] Mary J. [Henninger-]Voss, 'Between the Cannon and the Book: Mathematics and Military Culture in Cinquecento

turning spherical objects on a lathe; such practices took their theoretical support from commentators like Lefèvre, who first used the example of a lathe to show how a semicircle, when turned around an axis, could produce a solid sphere.[65] On his first reading, Beatus passed by the passage. But he did not forget the lesson. In a second copy of Sacrobosco that he bought in 1504, he corrected the commentary of Pedro Ciruelo by saying that 'This definition of the sphere, given by Euclid, is rather a craftsmanlike fashion of making a sphere, so it should rather be called a "description"'.[66] Glareanus, when commenting on the equivalent passage, gave the reason Euclid's account of a moving semi-circle could be called a definition: 'Let one imagine that this semicircle [i.e. the one Glareanus drew in the margine] be turned around an an axis; which should be a sphere, as in the first causal definition'.[67] What Beatus thought of as an example of craft, Glareanus thought of as a causal explanation. Beatus' first Sacrobosco, by Lefèvre, included the image of a lathe, which was reproduced in Paris through the 1530s, and then became standard fare in the editions of the Wittenberg printer Joseph Klug after 1532, as well as in editions that imitated him in Paris and elsewhere. Some readers of other editions even copied images of a lathe into the margins of textbooks lacking the figure: the 'craftsmanlike' analogy of the lathe became a topos in the genre.

[insert Figure 2 here]

Figure 2. *Opus novum astronomicum Jacobi Fabri Stapulensis*, ed. Christianus Sculpinus (Cologne, 1516), Newberry VAULT Ayer QB41 .S12 1508 no. 1, A4v. The manuscript image most resembles those printed in Wittenberg editions after 1531.

Renaissance students like Beatus and Glareanus were taught to read actively. They learned that 'difficult words, or matters of speciall obseruation … should be marked out … with little lines vnder them, or aboue them, or

---

Italy', Ph.D. dissertation (Johns Hopkins, 1995).

65 Lefèvre d'Étaples, *Textus de Sphera* (Paris, 1495), a4r. On the phenomenon generally, see Klaus Maurice, *Sovereigns as Turners* (Zurich, 1985); Horst Bredekamp, *The Lure of Antiquity and the Cult of the Machine. The Kunstkammer and the Evolution of Nature, Art and Technology*, trans. A. Brown (Princeton, 1995); Alexander Marr, *Between Raphael and Galileo: Mutio Oddi and the Mathematical Culture of Late Renaissance Italy* (Chicago, 2011), 155-9.

66 Ciruelo, D'Ailly, and Sacrobosco, *Uberrimum sphere mundi commentum*, BHS K950, a8v. 'Hec spherae diffinitio ab Euclide Megarensi assignata, magis fabricandae spherae modum industriam quamlibet, quare potius descriptio dicenda est'.

67 Glareanus' Sacrobosco, a2r. 'Imaginetur quispiam hunc semicirculum circum axem volui et sit spaera que ut prior diffinitio causalis est'.

against such partes of the word wherin the difficulty lieth, or by some prickes, or whatsoeuer letter or marke may best helpe to cal the knowledge of the thing to remembrance'.[68] Readers commonly learned three tactics to mark up their books: manuscript titles for key sections; short definitions and synonyms for words (sometimes scribbled between the lines); and finally blocks of commentary on passages.[69] So too in mathematical books. Readers of Sacrobosco underlined, sketched manicules, and flagged key words or arguments interlinearly and in the margin. They wrote out long scholia, sometimes from other books, sometimes from lectures. And they also drew diagrams and other figures—as I will discuss in the next two sections, this is perhaps one of the most distinctive features of mathematical reading.

The bulk of notes in copies of Sacrobosco reflect the habits of reading learned in grammar schools. In this mode, Beatus Rhenanus annotated the ways his master's commentary followed the standards of classical rhetoric, jotting down the names of authorities from George of Trebizond to Archimedes.[70] In many cases, a little learning in astronomy was simply another aid to understanding the literature of antiquity. What appeared to be a distinctively astronomical form of reading numbers and tables turned out to serve literary goals. The commentaries of Lefèvre especially encouraged this approach to the quadrivium, thickly supported as they were with tables of the longitudes and latitudes of antique and modern cities. (Lefèvre took advantage of the newly available *Geographia* of Ptolemy.) On Sacrobosco's account of the difference between 'unequal' days (measured by the sun, so variable in length throughout the year) and 'artificial' days (those measured by a clock, so always equal), Glareanus found the opportunity first to explain how different parts of Europe counted their hours either from sundown or sunrise, before expanding on how the Gospel of Matthew counted from sunrise in Jesus' parables.[71] Other readers of Sacrobosco frequently expected that their modicum of astronomical knowledge would serve other literary ends. In a copy now in the Huntington Library, a reader copied out large sections of Cicero's legal oration *In Rullam*, drawing lines to link

[68] John Brinsley, *Ludus literarius; or, The Grammar Schoole* (London, 1612), qu. in Sherman, *Used Books*, 3–4. Erasmus of Rotterdam gave instructions for annotating in his 'De ratione studii', published in *Collected Works of Erasmus* 2, ed. Craig R. Thompson (Toronto, 1978).

[69] These heuristic categories are also found by Grafton and Leu, *Henricus Glareanus's (1488-1563) Chronologia of the Ancient World*, 19. See more generally Sherman, 'What Did Renaissance Readers Write in Their Books?'

[70] Lefèvre, *Textus de Sphera* (Paris, 1500), BHS K1046c a[1]v. Beatus noted Lefèvre's first lines as a 'Captatio benevolentiae'.

[71] Glareanus' Sacrobosco, B3r. 'Veluti et horae euangelicae et horae passionis Christi inchoantur ut Mathaei 12, de paterfamilias mittens operarios in vineam suam, nonnullos hora primum, hoc est ortu solis, per secundum, duas horas post ortum intelligit per undecimam unam autem solis occasum intelligit. Quod ex fine parabolae depraehendis. Missi namque hora undecima operaverunt solum ad unam horam'.

these quotations to cities that Cicero mentioned such as Ligura, Capua, and Carthage.[72] Sacrobosco, as supplemented by these sixteenth-century editors and commentators, thereby became a literary tool, providing a reader further details to place the lives of ancient authors. Similarly, on the flyleaf of the copy now in the Huntington Library, the reader quoted the newly available work of Vitruvius, *De architectura* 9.2, comparing the ancient 'Chaldean theory' of Berosus on the moon's phases with that of Aristarchus of Samos.[73] (Berosus thought the moon comprised a light and dark hemisphere, moved by the sun; Aristarchus argued that the moon circled the earth.)

The literary arrow could go the other way too, when literature clarified astronomy. Much reading of astronomy, as of other early modern genres, was soaked with proverbs and ancient *bon mots*, often added to the titlepage as a kind of studious motivation. Beatus remarked that 'the intellect's edge augments the power of sensible instruments'.[74] Glareanus contented himself with a line that summarized astronomy as a discipline: 'Astronomy is the correct law and rule that looks to the magnitudes and motions of the heavenly bodies above'.[75] Sacrobosco himself had turned to Ovid, Lucan, and Virgil to shed light on the workings of the heavens and the character of the different climes.[76] Readers frequently adorned the titlepages with snippets of insight that sometimes stretched into fulsome quotations. One reader of a Wittenberg *Sphere* from 1540 copied out on the flyleaf ten lines from Ovid's *Metamorphoses* on the subject of the 'double motion" of the heavens.[77] Under the right conditions, letters not only

[72] Sacrobosco, *Sphera cum commentis* (Venice, 1508), Huntington, Burndy 751765, 61r. Lecture notes from 1573, by John Chambers at Oxford, show extensive use of Cicero and other literary sources. These notes now partially preserved in Oxford, Bodleian Library MS Savile 30. Cit. Robert Goulding, 'Testimonia Humanitatis: The Early Lectures of Henry Savile', in Francis Ames-Lewis (ed.), *Sir Thomas Gresham and Gresham College*, Studies in the Intellectual History of London in the Sixteenth and Seventeenth Centuries (Aldershot, 1999), 143–4. Compare also Sacrobosco, *Sphera cum commentis* (Venice, 1508), Huntington 497564, 56r, where the reader reflects on the eclipse seen in Jerusalem at Christ's crucifixion.

[73] Sacrobosco, *Sphera cum commentis* (Venice, 1508), Huntington, Burndy 751765, end guard page. Note that this reader would not have found Aristarchus' heliocentrism in Vitruvius. It is reported in Archimedes' *Sandreckoner* and in Plutarch.

[74] Beatus' Sacrobosco, titlepage. 'Provi.| Acuties intellectus quorumcumque sensibilium instrumentorum virem supplet'.

[75] Glareanus' Sacrobosco, title page. 'Astronomia est recta lex et regula superiorum corporum magnitudines et motus considerans'.

[76] E.g., Thorndike, *The Sphere of Sacrobosco and Its Commentators*, 87, 91, 94, 95, inter alia. Readers commonly marked these passages: e.g. Pedro Ciruelo, *Uberrimum sphere mundi commentum* (Paris, 1498), Newberry folio Inc. 8015, e.g. h4r-5r. In some cases they copied further lines from the ancient author into the margin: Sacrobosco, *Libellus de sphaera* (Wittenberg, 1538), BL 417.c.4, B5r.

[77] Sacrobosco, *Libellus de sphaera* (Wittenberg, 1542), Houghton *GC.04625.474tm, flyleaf. The reader titled the quotation from *Metamorphoses* 2.63-2.73, 'Ovidius de duplici motu'.

drove students to the text, but the poetic muse stirred within astronomy itself. The same reader who turned to astronomy to enlighten his reading of Cicero also penned his own lumbering lines in dactylic hexameter on the 'True Worship of God', exploring the relationship between humanity as the temple of God. To remind himself of the meter, he added the first line of Virgil's *Aeneid*, branded so deeply in every schoolboy's memory as the archetype of dactylic hexameter: 'Arma virumque cano, Troye qui primus ab oris'. He then professed his faith with a hymn, in the same meter, on the structure of creation: 'Amo deum qui me creavit ex nihilo in mundo'.[78]

Astronomy and other mathematical works were read alongside logical works. More systematic readers strategically noted the logical and rhetorical moves made by Sacrobosco's commentators. In the late 1580s, an otherwise unknown Nicholas Gavius reflected the common Renaissance debates over rhetorical and logical order by noting the commentator's choice to first describe Archimedes as the inventor of spheres, while naming Sacrobosco as the chosen authority of Parisian scholars. As he pointed out, this is a matter of 'quid est' (the given topic or observation), which properly proceeds discussion of causes, 'quia est'.[79] This particular reader, like many others, chose only to analyze the rhetorical opening of the commentary. Some displayed much more stamina in their commitment to parsing mathematical discourse according to the standards of school logic: one of the three readers of Burndy 751765 (now at the Huntington) annotated and underlined hundreds of 'major' and 'minor' premises within Esculano, Capuano, Lefèvre, and d'Ailly.[80] At first this suggests only the mundane observation that university logic was everywhere, so that even in reading mathematical works the schools encouraged a thoroughly unmathematical activity through endless detours into rhetoric and dialectic—precisely what critics of the status quo such as Henry Savile decried as the ruin of mathematical education.[81] But at the same time, when considered as part of the history of mathematics, such dedication to logical analysis of astronomical reasoning is a reminder that the shifts in rhetoric and logic that characterized Renaissance pedagogy had implications for the quadrivium as well. This is not a minor point, since the sixteenth century was a period in which the relative certainty of logic and

[78] Sacrobosco, *Sphera cum commentis* (Venice, 1508), Huntington, Burndy 751765. Many copies of this edition bear short lines of poetry on their titlepages: BL IA.40072; Newberry VAULT Ayer 6 .S2 1507.
[79] Lefèvre, *Textus de Sphaera* (Paris, 1516), Houghton GEN typ 515.16.764, a4r. 'quid est precedit quia est'.
[80] A similar case is Sacrobosco, *Sphera cum commentis* (Venice, 1508), Huntington, Burndy 751765. There are systematically blocked out sections in Pedro Ciruelo, *Uberrimum sphere mundi commentum intersertis etiam questionibus domini Petri de Aliaco* (Paris, 1498), Newberry folio Inc. 8015. Readers often presented their notes in the argumentative form of *questiones* or *dubia*. E.g.: Lefèvre, *Textus de sphera* (Paris, 1495), Huntington 105170.
[81] Goulding, 'Testimonia Humanitatis'.

mathematical reasoning—not seen as the same thing—were in flux.[82]

Studying astronomy alongside other disciplines, readers often reflected on the status of astronomy and its relationship to other domains of knowledge. Glareanus especially commented on the difference between poetic and astronomical kinds of truth. 'Our author speaks according to the opinion of poets, who call the torrid zone this [i.e. uninhabitable]', he observed.[83] As Vespucci's experience of travel all the way to the antarctic proved, it was possible to live beyond the bounds of the torrid zone. But Glareanus suggested that—unlike astronomers—poets could be allowed some license: 'that you call "uninhabitable" a zone which can hardly be lived in, so the poets affirmed the truth. For not many live under the torrid zone'.[84] By distinguishing poetic *sententia* from the precision of astronomers, Glareanus at once affirmed and limited the value of literature for astronomy. The poets should be read in light of current experience, not the other way around.[85]

Certain questions seem to have become a standard *topos* for delineating astronomy from other domains. A popular question arose around Sacrobosco's claim that the universe comprised nine spheres, the ninth sphere having been introduced by astronomers to account for the precession of the equinoxes. Philosophers noted the discrepancy with Aristotle, adding a tenth sphere. Theologians sometimes identified this tenth sphere as the *empyrean*, fixed in medieval astronomy on the authority of Peter Lombard's *Sentences*.[86] In fact, at least at Paris, it appears that the tenth sphere had been a question of faith since the thirteenth century. Authorities such as Bonaventure or Aquinas all supported the presence of the empyreum on the strength of 'probable arguments' drawn from the Bible; some, such as Pierre d'Ailly, tried out physical arguments as well, such as whether this extreme, immobile heaven could

---

[82] Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford, 1996); Timothy J. Reiss, *Knowledge, Discovery and Imagination in Early Modern Europe* (Cambridge, 1997). The relevance of logic and rhetoric for mathematical practice deserves further consideration, following work by Giovanna Cifoletti, 'From Valla to Viète: The Rhetorical Reform of Logic and Its Use in Early Modern Algebra', *Early Science & Medicine* 11 (2006), 390–423.

[83] Glareanus' Sacrobosco, B1r. 'Auctor noster loquitur secundum poetarum sententiam, qui torridum eam dixerunt'.

[84] Ibid. 'inhabitabilem diceris zonam quae vix inhabitatus potest, hinc verum poetae dixerant. Nam non multi sub torrida zona habitant'. On contemporary debates about the habitability of the torrid zone, see Christine R. Johnson, *The German Discovery of the World: Renaissance Encounters with the Strange and Marvelous* (Charlottesville, VA, 2008), 64–71.

[85] This stance holds interesting implications for Glareanus' view of scripture, because elsewhere he quotes the Psalms as evidence for the earth's stability, citing the 'regius poeta'. According to Glareanus, does scripture deploy the same *sententia poetarum* as Virgil, and so speak less precisely than astronomers?

[86] The emergence of the Empyrean and the tenth sphere is discussed in detail by Edward Grant, *Planets, Stars, and Orbs: The Medieval Cosmos, 1200-1687* (Cambridge, 1994), chap. 15, especially 376, 378-382.

influence lower spheres (though d'Ailly did accept the empyreum).

Sixteenth-century students learned about the empyreum as a case in navigating authorities. Beatus Rhenanus included a long paragraph on the number of spheres, drawing on King Alphonse (of the Alphonsine Tables, which turned the Spanish king into an authority), Peurbach, Regiomontanus, and other *recentes*. 'Beyond these heavens known by nature [i.e. the seven planetary orbs and the stellar sphere], the sacred theogians posit another heaven, the empyreum, which is fiery'.[87] But Beatus' presentation of the theological argument is less than convincing, and he gives most space to the physical argument against it noted by d'Ailly. The theologians do not argue for the empyreum by naturally reasoning about its properties, he reports, but on account of its spendor—and as it is the seat of God, they insist it must be immobile. But that does not make sense, according to Beatus, because if it were immobile, how would it influence the lower spheres, which plainly do move? If the outermost sphere were still, would it not slow inner spheres down? 'But against this others argue that the influence of this ruling sphere does not pertain to the order of the bodily world, but only to the matter of this heaven, and so it is known not by natural means but by divine contemplation'.[88] The argument from faith considered, Beatus concludes with the arguments of astronomers and natural philosophers: 'modern astrologers', he pointed out, 'see that the fixed stars (the eighth sphere) has a three-fold motion. Thus they conclude that two other spheres should be given beyond the eighth sphere'.[89]

In a later note that returned to the topic, Beatus presented the problem as unsolved, and all options as open. He began by describing the differences between spheres, as properly solid objects, and orbs, as hollow objects. 'But these nine, properly called orbs, are commonly called spheres'.[90] Yet the number was not certain: 'But the number, in this matter, is undecided by philosophers, for according to some, there are eight; according to others, nine, like

[87] Beatus' Sacrobosco, a1v. 'Preter hos celos naturales cognitos, sacri theologi celum aliud empyreum, hoc est igneum, ponunt'.
[88] Beatus' Sacrobosco, a1v-a2r. 'econtra autem alii influxum huius spherae regens ipsam dicunt non pertinere ad ordinem mundi corporalis sed huius celi tractatum, eo quod non via naturali sed sola fide et divina contemplatione cognoscitur'.
[89] Beatus' Sacrobosco, a2r. 'Moderni autem astrologi … viderunt stellas fixas sive octavam spheram triplicem motum habere, que concludunt duas alias spheras esse dandas preter octavam spheram'.
[90] Beatus' Sacrobosco, a3v. 'Et hae proprio novem dicuntur orbes communi vero spherae'. Beatus defended the common terminology as an example of the species sometimes taking on the name of the higher genus it comes from, citing the *Topics* I, and the examples of 'dispositio' used to mean both 'dispositus' and 'habitus', and 'casus' used to mean both 'casus' and 'fortuna', as described in *Physics* II.

this author [of the *Sphere*]; according to yet others, ten'.[91] The details here were not important. Instead, what mattered was whether the *Sphaera mundi* was properly categorized. Beatus defended the common terminology as an example of the species sometimes taking on the name of the higher genus it comes from, citing the *Topics* I, and the examples of 'dispositio' used to mean both 'dispositus' and 'habitus', and 'casus' used to mean both 'casus' and 'fortuna', as described in *Physics* II.[92] In this case, rather than learning the distinctions between how philosophers and theologians reasoned about the numbers of spheres, Beatus ended the scholium with the encyclopedic question of considering whether in fact the *Sphere* was about spheres. He (or his master) reached the conclusion that it was, since the *Sphaera mundi* dealt with the universe not as a sphere 'taken simply, but rather by aggregation'.[93] This emphasis reveals Beatus less interested in astronomical arguments themselves as in checking a box on his chart of the disciplines.

Readers often were concerned to set up the proper boundaries between 'natural' reasoning and the authority of theologians. One exemplar from Cracow includes leaves of notes sewn in to the binding, which supplement the dense marginal notations. Writing sometime after 1522, the student describes three opinions on this question: that of the philosophers, who claim only eight spheres; the astronomers' view, who follow Ptolemy's account of nine spheres, including the seven planets, one in which the stars are embedded, and a ninth to account for the slow procession of the equinoxes; and finally the view of theologians, who posit a tenth sphere, calling it the 'caelum empireum'. The Cracovian reader cited Alphonse of Spain among his sources, and followed a line of reasoning nowhere hinted in the text before him by John of Glogovia (c. 1445-1507). In particular, he observed that one might wonder 'beyond nature" (*preter naturam*) about the source of the ninth sphere's motion. Clearly not from the spheres below, given Aristotle's point that in heavenly motion, the larger spheres possess greater motion and therefore must transmit it downward. Thus the ultimate source of motion, he implied, must be from the fiery

[91] Beatus' Sacrobosco, a3v. 'Quot autem sint secundum subiectam, numerus apud philosophos est indeterminantus: secundum aliquos enim sunt Octo; secundum alios, Novem, ut secundum autorem de sphera; secundum alios decem'.

[92] Beatus' Sacrobosco, a3v. 'Hoc etiam modo dispositio dividitur in dispositum et habitum, et casus dividitur in casum et fortunam, 2o *Physicorum*; et respublica in regnam, optimorum gubernationem, rempublicam popularitatem, paucorum potentiam, Tyranidem et alias rei publicae species in 3o *Politicorum*'.

[93] Beatus' Sacrobosco, a3v. 'universum est una sphera non quidem simpliciter accipiendo unum sed per aggregationem'.

residence of God, and the tenth sphere must exist.[94] While deferring to theologial authority, this reader, like Beatus, focused on the transmission of motion between the orbs of planets to reason about their relations—but with the opposite conclusion, that the motion of the lower spheres suggests the existence of the empyreum.

Theological authority frequently motivated a reader's marks. Remarks often celebrated where the natural world reflected divine truth. Glareanus pointed out that the centrality of the earth seems to be assumed by the author of the Psalms: 'David, the poet king: "Who laid the foundations of the earth; it shall never be moved" [Ps 103:5]. In another place it says, "he founded it upon the seas, and he set it on the waters" [Ps 24:2]'.[95] Other readers moved away from this devotional mode to polemics, expressing outrage at offensive ideas. In most cases, however, they faulted the commentators on the *Sphere*, not Sacrobosco himself. One of the readers of the 1513 Cracow edition in Owen Gingerich's collection crossed out whole paragraphs. In one particularly emphatic act of effacement—doubly crossed out—this reader evidently frowned on the impertinence of the author, John of Glogovia, against the theologians who claimed that Jerusalem is the center of the world.

[Insert figure here]

Figure 3. John of Glogovia, *Introductorium compendiosum in tractatum spere materialis* (Cracow, 1513), private collection Gingerich, a8r. Beside a censored passage about Jerusalem's location, this page bears annotations in two hands, including examples of geometrical structures.

John's text showed that the claim made no sense, according to the principles of the *Sphere.* Sacrobosco observed that if Jerusalem were at the center of the world, those living there would experience the unchanging length of days (as at the equator), rather than a seasonal shift. But the reader evidently was conconvinced, and found this inference impious enough to deserve censure. Other traces of religious pressure are less innocuous. Later in the sixteenth century readers defaced their books to hide the confessional affiliations of their contents. It is common to find exemplars of Sacrobosco with 'Wittenburg" scratched and scribbled out or pasted over, along with the name 'Melanchthon'. Melanchthon's essay prefacing the *Sphere* became an enormously popular exordium to

[94] Joannis Glogoviensis, *Introductorium compendiosum in tractatum spere materialis* (Cracow, 1513), private collection of Gingerich, sheet tipped in between a3–a4. On the proper motion of heavenly spheres, see Aristotle, *De caelo* II.8, inter alia.

[95] Glareanus' Sacrobosco, a4r. 'David Poeta regius: Qui fundasti terram super stabilitatem suam non inclinabitur in saeculum saeculi. Alio in loco dicitur super maria fundavit eum et super fluviam preparavit eum'.

mathematical study, and even colleges of Catholic religious flouted censorship to include it.[96] Censorship and efforts to evade censorship can be traced to Protestants as well; in one copy in the Huntington Library, a reader has carefully pasted over every single mention of Pierre d'Ailly, presumably for his actions against the Hussites at the Council of Constance (1514-1518).[97] Yet censorship mostly extends only to names, not text itself. Such marks suggest that in midst of confessional tensions, and despite the carefully-maintained disciplinary boundaries between theology and natural philosophy, some readers thought that knowledge of nature could be detached from the lives of authors—nature as a source of non-confessional knowledge—though they remained acutely aware of how their reading could be perceived by religious authorities.[98]

The great majority of annotations on the *Sphere* represent ways of study and reading deployed across the curriculum. Students considered the rhetorical and logical peculiarities of the text before them, as well as of its commentators, and as they did so, they integrated Sacrobosco into their larger picture of the universe. This kind of reading did not necessarily flatten out the differences between astronomy and other disciplines; instead, students learned to recognize their contours, and to set them in a hierarchy.

2. Comparison of Commentaries and Authorities

In the previous section, I surveyed a range of marking strategies that readers of Sacrobosco might have applied to any sort of school text. This section turns to a related practice that is important enough to deserve a category of its own: the comparison of different texts, authorities, and editions. Historians of science have begun to see how reading practices could enrich technical works. Renaissance readers learned how to excerpt, to recycle bits of information into commonplace books, to update their books in the margin, and to compile such notes into reference works that could then be again updated.[99] For instance, in natural history the discoveries of the New World entered

---

[96] Sacrobosco, *Libellus de sphaera* (Wittenberg, 1542), Houghton Library *GC.P4625.47tm, titlepage. The titlepage also indicates Capuchin ownership: 'Ad usum F.F. Capuccinorum Buhani 1666'. Many copies indicate Jesuit ownership; nearly half the copies I have seen bear an *ex libris* from religious houses.

[97] Sacrobosco, *Sphera cum commentis* (Venice, 1508), Huntington, Burndy 751765.

[98] Such readership evidence might balance narratives that stress the polarization of confessional natural philosophies, such as Sachiko Kusukawa, 'Nature's Regularity in Some Protestant Natural Philosophy Textbooks 1530-1630', in *Natural Law and Laws of Nature in Early Modern Europe: Jurisprudence, Theology, Moral and Natural Philosophy*, ed. Lorraine Daston and Michael Stolleis (Aldershot, 2008), 120–1.

[99] E.g. Ann Blair, 'Annotating and Indexing Natural Philosophy', in M. Frasca-Spada and Nicholas Jardine (eds.) *Books and the Sciences in History* (Cambridge, 2000), 69–89. One of the most remarkable instances of such practices is the *pandechion epistemon* of Ulisse Aldrovandi: see Fabian Krämer, 'Ein papiernes Archiv für alles

the common knowledge of Europeans only slowly—these discoveries were absorbed into an intellectual culture deeply committed to learned textual practices.[100] Similarly, as with the examples of lathe figures discussed above, the new printed textbooks of astronomy standardized images and commentaries, and readers compared diagrams and arguments across editions of Sacrobosco.

The expanding collections of commentaries and supporting texts also made it possible to compare differing authorities within the volume. Not all readers were as direct as Johannes Sebulerus [?] of Thuringia (otherwise unknown), who delivered a verdict on the titlepage of his copy: he denounced the fourteenth-century astrologer Cecco d'Ascoli as 'dreaming and raving', while Capuano de Manfredonia was, he allowed, 'a man of excellent intellect'.[101] I have already remarked how Peurbach was frequently mentioned by readers annotating discussions of the number of spheres. Commentaries printed together, sometimes in parallel, introduced a kind of fluidity between books and editions, so that a reader might work back and forth until the matter at hand was sufficiently clear.[102]

Many annotations reveal the reader in mental dialogue with another book. For example, in Pedro Ciruello's commentary on the *Sphere* Beatus Rhenanus pointed out where Ciruello echoed Lefèvre's distinctive argument—'Lefèvre d'Étaples is of this view'.[103] Such markings give a sense for who populated the intellectual universe of the readers: not only did Beatus note the names of typical astronomical authorities like Regiomontanus and Peurbach, but on the endpapers of the *Sammelband* and several times in the margins he mentioned Dionysius the Areopagite,

jemals Geschriebene: Ulisse Aldrovandis Pandechion epistemonicon und die Naturgeschichte der Renaissance', *Zeitschrift für Geschichte der Wissenschaften, Technik und Medizin* 1 (2013), 11–36.

[100] Anthony Grafton, April Shelford, and Nancy G. Siraisi, *New Worlds, Ancient Texts: The Power of Tradition and the Shock of Discovery* (Cambridge, MA, 1992). Albrecht Dürer's rhinoceros is a classic case, recently described by Susan Dackerman, 'Dürer's Indexical Fantasy: The Rhinoceros and Printmaking', in Susan Dackerman (ed.) *Prints and the Pursuit of Knowledge in Early Modern Europe* (New Haven, 2011), 164-84. The slowness of this process, enduring into the eighteenth century, is evident in Neil Safier, *Measuring the New World: Enlightenment Science and South America* (Chicago, 2008). On natural history specifically, see Brian W. Ogilvie, *The Science of Describing: Natural History in Renaissance Europe* (Chicago, 2006). A similar point is made by Jed Z. Buchwald and Mordechai Feingold, *Newton and the Origin of Civilization* (Princeton, 2012).

[101] Esculano, Capuano, and Lefèvre d'Étaples, *Sphera mundi cum tribus commentis* (Venice, 1499), private collection of Owen Gingerich, titlepage: [of Ciccho Esculano] 'somnantis et delirantis'; [of Capuano] 'excellentis ingenii viri'.

[102] For one example of cross-referencing within a collection, see Ciruelo, *Uberrimum sphere mundi commentum* (Paris, 1498), Newberry folio Inc. 8015, b8r. Early modern readers valued editions that allowed them to compare commentary and text traditions—a format which goes back to the early Bible translations of Origen and Jerome—as can be seen in polyglot editions of the Bible, but also side-by-side translations of Euclid: Campanus and Bartholomaeo Zamberti, *Euclidis Megarensis Geometricorum elementorum libri XV* (Paris, 1517).

[103] Ciruelo, *Uberrimum sphera mundi commentum* (Paris, 1498), BHS K950, b1r. 'Huius opinionis est Faber Stapulensis'.

the medieval Catalan Ramon Lull, and Nicholas of Cusa.[104] Glareanus noted various arguments from Ptolemy, with great respect, but accorded special attention to Vespucci's 'Cosmographia', and also alerted his students to comparable passages in Gregor Reisch's *Margarita philosophica*.[105]

Indeed, Sacrobosco allowed students to harmonize a broad intellectual context—and many students were curious about astronomy's links to astrological or even magical power. Where Sacrobosco divided the sphere of heaven into the various poles and circles, Lefèvre noted that 'magicians in particular identify four points', and then listed them as the point of God (east), of intelligences (midday), of the blind (west), and evil powers (midnight).[106] Beatus Rhenanus's student notes took the question much further. He announced the topic by listing 'Prophetae, Magi, Philosophi', and then clarified their hierarchy: 'Magicians are above philosophers, being those who by skilful searching dig out the secret and hidden matters of nature herself. Magic is the science by which one enters the mysteries of nature. Our own Giovanni Pico della Mirandola in particular brought light on this matter'.[107] Thus astronomy offered tools for a Platonic conception of universe filled with whispers of divinity. Using a table to structure the four points and their 'analogies' (Figure 4), Beatus brought the discussion to a close by bringing together Plato and the Bible. First he noted that 'the ideas of all perfect properties are in God'.[108]

[insert figure here]

Figure 4. Lefèvre, *Textus de sphaera* (Paris, 1500), BHS K 1046c (Beatus' Sacrobosco), a8r. Below the discussion of 'Prophetae, Magi, Philosophi', Beatus includes a table correlating the four points of heaven to

[104] For example, see Beatus' notes on the first flyleaves his *Sammelband* of mathematical textbooks, BHS K1046, where he notes the pyramid or cone as an analogy for the scale of being.

[105] Glareanus' Sacrobosco, e.g. A4r (references to the Bible, Ptolemy, Gregor Reisch); A5r (Aristotle, *De gen. et core.*); B1r (Vespucci, from Ringmann's *Cosmographia*); B3r (the Bible); B6v (Vespucci from *ibid*., Ptolemy).

[106] Around the time that he published this commentary in 1495, Lefèvre also wrote a manuscript work *De magia naturali*, which described magic as the 'practical part' of natural philosophy. Within a few years, he spoke much more carefully about magic as deceptive. For the most recent evaluation of Lefèvre on magic, see Jean-Marc Mandosio, 'Le De magia naturali de Jacques Lefèvre d'Étaples: Magie, alchimie et cabale', in *Les Muses secrètes : kabbale, alchimie et littérature à la Renaissance*, ed. Rosanna Camos Gorris (Geneva: Droz, 2013), 37–79.

[107] Beatus' Sacrobosco, a8r. 'Magi sunt super philosophos et sunt qui solerti indagine ad secreta et arcana ipsius naturae investigant. et magia scientia est qua naturae mysteria ingreditur. In qua inter nostrates Ioannes Picus Mirandula maxime eluxit'. Note that Beatus (and presumably his master) holds Pico as an authority in this area, even though it must have been common knowledge that Pico denounced astrology in his *Disputationes adversus astrologiam divinatricem* (Bologna, 1496). On the reception of Pico, see Steven Vanden Broecke, *The Limits of Influence: Pico, Louvain, and the Crisis of Renaissance Astrology* (Leiden, 2003), 55–80.

[108] Beatus' Sacrobosco, a8r. 'In deo sunt ideae omnium perfectarum proprietatum'.

their analogies.

Then he repeated the biblical quotation with which Lefèvre justified his analogies: ‘for the invisible things of God, as the apostle says, are seen from the creation of the world, understood through those things that have been made’.[109] Other readers found similar associations in Sacrobosco. Thomas Corser, whose notes identify him as a late sixteenth-century presbyter, while reading Lefèvre’s comments on the universe as *machina mundi* was reminded of Cicero’s account of the world’s rational order.[110] And then a few pages later Thomas was reminded by Lefèvre’s comment on how magicians organize the four points of the zodiac of a specific book on natural magic, the *Arbatel* linked to Cornelius Agrippa.[111]

Since the *Sphere* was the most basic of textbooks, it is at first surprising that readers of Sacrobosco also brought in scholarly, textual comparisons. Such learned analysis seems overzealous; but Paris editions from 1520-38 as well as the increasingly popular versions from Wittenberg, complete with Melanchthon’s preface, suggest that mathematics belonged to the well-rounded man of letters, because they included printed marginalia on Greek phrases and references to Greek spherical geometry and newly available authors like Proclus and Hesiod. A copy of a Wittenberg edition of the *Sphere*, now at Houghton Library, exemplifies the kind of reading such classical trappings could elicit. Probably in the 1540s, the reader noted that Sacrobosco was working from the Campanus translation of Euclid’s *Elements*, and went on to verify Sacrobosco’s interpretation of Theodosius’ treatise on spherical geometry.[112] Throughout the pages are short passages from Aristotle in Greek, indicating the reader’s ability and desire to integrate this medieval textbook with his classical authorities.[113] While the level of scholarship

[109] Beatus’ Sacrobosco, a8r. ‘Invisibilia enim dei (ut inquit Apostolus) a creatura mundi per ea que facta sunt intellecta conspicitur’. See Romans 1:20.

[110] Lefèvre, *Textus de Sphera* (Paris, 1527), Houghton, f EC.Sa147s.1527, a5r. ‘4or secundum Cicerone lib. de natura deorum 2o’. Likely the intended reference was Cicero, *De natura deorum* II xxxvii.97: ‘An, cum machinatione quadam moveri aliquid videmus, ut sphaeram ut horas ut alia permulta, non dubitamus quin illa opera sint rationis’. Similarly, the marginalia of Sacrobosco, *Libellus de sphaera* (Wittenberg, 1538, BL 417.d.4, B4v, B5r.

[111] Lefèvre, *Textus de Sphera* (Paris, 1527), Houghton, f EC.Sa147s.1527, b1r. ‘de magica descript. Cael Cardin. vid. Cornel. Agrip. Arbatel, etc’. Spuriously attributed to Agrippa, probably edited by Theodore Zwinger, *Arbatel* was first published in 1575 (Basel).

[112] Sacrobosco, *Libellus de sphaera* (Wittenberg, 1542), Houghton *GC.P4625.474tm, at cap. 1. ‘In definitioni Euclides sumpta ex translatione Campani describa cum formalem spherae quo fiat’. | ‘In theodisii tales est in principio Arg’.

[113] Houghton *GC.P4625.474tm (B2r), in which a reader quotes from a Greek edition of *De caelo et mundo*, giving two definitions of *kosmos*.

is low, it does indicate the company Sacrobosco was expected to keep—successfully, such notes suggest, at least for a while.

Many, perhaps even most, annotations that compare Sacrobosco with various authorities are not specifically mathematical. Like Glareanus' use of Vespucci to consider whether the sub-tropical zones are habitable, such notes weigh the testimony of different authors; like the anonymous marginalia identifying the longitudes and latitudes of cities Cicero had mentioned, such notes put mathematics to work clarifying other texts, but do not test the technical issue at play. But occasionally marginalia in Sacrobosco reflect a distinctively mathematical form of reasoning, particularly in diagrams and figures.

The importance of diagrammatic thinking to astronomy is especially obvious when readers seized on particular images and copied them into their own books. Glareanus performed this service for his students in his own Sacrobosco with particular care, most likely drawing on at least three other editions of Sacrobosco. One image reproduces iconography found in the many Venice editions, first illustrated by Erhardt Radtholt in 1485 and quickly expanded upon by J. L. Santritter and H. de Sanctis from 1488. From this iconography Glareanus took at least three elements: a cross-section of the four elements within the aetherial spheres of heaven[114] (Figure 5a); a circle with an inscribed square and triangle to demonstrate that observations of the heavens only harmonize with a spherical description of earth's surface[115] (Figure 5b); and a ship that bears two sailors, to support an argument for the earth's rotundity[116] (Figure 5c; since only the higher sailor can see a point on land, the earth must bulge to block the lower sailor's view).

[insert figures here]

Figures 5a, 5b, 5c. Details from Joannes Sacrobosco, *Tractatus de Sphera* (Paris, 1493), Munich University

[114] I suggest this because the iconography for air is quite distinct, with a undulating line commonly used elsewhere to represent clouds. Glareanus' representation of air closely matches that found in the Venetian tradition of these diagrams.

[115] This image is typical in the Venetian tradition, but not found in either of the other two books Glareanus copies illustrations from (as described below).

[116] Versions of this image were widely available in both manuscript and printed copies of Sacrobosco (e.g. MS Ambrosiana C 241 Inf., 152r). however, the illustration in Glareanus' other edition of the *Sphere* does not have a tower, the ship faces the other direction, and the earth's bulge is hardly prominent. Although it is possible that Glareanus coincidentally modified his own drawing to match the widely available Venice editions, I think it more likely that this is a visual quotation.

Library 4 Inc. Lat. 310#6 (Glareanus' Sacrobosco), a2v, a3r, and a3v. Images from this Venice tradition of illustration are discussed by Crowther and Barker, 'Training the Intelligent Eye'.

Further images clearly come from the other, sparcely illustrated, copy of Sacrobosco in Glareanus' possession: notably, a pair of diagrams of two small people at different places on earth (Figures 6a and 6b). In the top diagram, the different horizons for the two figures are neatly measured and placed alongside each other, to show the different views of the heavens available to observers at different longitudes on earth. Similarly, in the second diagram, the two people are at different latitudes; the one is 'seeing stars' (*videns stellas*) and the other 'not seeing stars' (*non videns stellas*).

[insert figures here]

Figure 6a. Sacrobosco, *Opus sphericum* (Cologen, 1505), C1r, detail.

Figure 6b. Glareanus' Sacrobosco, A3v, detail.

Glareanus reorients the illustrations to fit in the margins, but otherwise copies them faithfully, down to the labels. His third source for illustrations strays beyond the *Sphere*, but not far: Ringmann's *Cosmographiae introductio*. One of the several figures Glareanus borrowed from the book makes clear that he had curated this collection of astronomical images for the sake of students who might borrow his book. 'Still, you see that, because of the lack of space, not everything is worked out with the greatest of clarity. But, dear reader, be satisfied enough with my work' (Figure 7).[117]

[insert figure here]

Figure 7. Glareanus' Sacrobosco, B6r, detail, copied from Ringmann, *Cosmographiae introductio* (St Die, 1507), a1v.

Given the care with which Glareanus copied such figures into his 1493 edition of Sacrobosco, he may very well have chosen to annotate an un-commented and—by 1507 rare—unillustrated older edition of the *Sphere* specifically in order to create a useful teaching textual and visual aid.

[117] Glareanus' Sacrobosco, b6r. 'Propter spatii tamen indigentiam non omnia luculentissime elaborata conspicis. Sed labore nostro lector contentus bene. Vale'.

Glareanus wrote his annotations as an example for others; his is perhaps a specially consistent case of harvesting figures from a wider reading in order to elucidate Sacrobosco's text. But this was not an unusual practice. A reader might link arguments throughout a text to a particularly helpful image, referring back to the same image throughout the volume.[118] More frequently, readers drew on a stock of images that had become standard in the various editions of the *Sphere* since the 1478 Venetian edition of Franciscus Renner, and when a particular topos came up without the image, they supplied it out of another edition. In particular, readers seemed to find compelling images of the stars moving in a circle above observers on earth.[119]

[insert figure 8 here]

Figure 8. Lefèvre, *Textus de sphaera* (Paris, 1521), BnF res.v.209, a6r. To explain the rotundity of heavens, the reader copied figures originally printed in earlier Venetian editions of Sacrobosco.

These images show the astronomical reader wading through a flood of book options, mixing and matching from different editions to find the 'standard' visual forms.[120]

As noted at the beginning of this section, historians of science have begun to see learned practices of reading as an important way that natural knowledge was collected and taught. The new collections of competing commentaries on the text of the *Sphere* became a site for applying these textual practices to astronomy. The result was that in the first decades of the sixteenth century the medieval structure of the *Sphere* grew heavy under thickening coats of antique paint. In many cases, these additional layers cement Sacrobosco more firmly in traditional perspectives, as when Beatus' evoked biblical motivations for the four astrological points of heaven; at the same time, however, such accretions also bring the enormous apparatus of two-sphere astronomy into dialogue with new sets of evidence, as when Glareanus repeatedly mentioned Vespucci's expeditions.

So too for figures. Cases like Glareanus' Sacrobosco show how readers encountered a specifically visual culture of astronomy. Not only did Sacrobosco's readers learn to integrate their textual cosmos through

---

[118] Esculano, Capuano, and Lefèvre d'Étaples, *Sphera mundi cum tribus commentis* (Venice, 1499), BnF res.v.199: [on image of c. mundi and c. eccentrici] 'pro hac lectione opertet videre theoricas planetarum georgii purbachii hec sunt in hoc volumine'.

[119] Besides Glareanus' Sacrobosco, these figure are repeatedly copied in Lefèvre, *Textus de sphaera* (Paris, 1521), BnF res.v.209, fol. 6r; Glogoviensis, *Introductorium compendiosum in tractatum spere materialis* (Cracow, 1513), collection of Owen Gingerich, B3v.

[120] On visualization, see especially Crowther and Barker, 'Training the Intelligent Eye'.

Sacrobosco's words, but they learned to reimagine the world visually by comparing and collating a distinct visual grammar. Isabelle Pantin, Kathleen Crowther, and Peter Barker have drawn attention to Kepler's appropriation of figures from the *Sphere* tradition in order to present Copernican astronomy in his *Epitome*; expositors of the new astronomy depended on the *Sphere*'s visual grammar in order to be understood.[121] I would suggest that Sacrobosco's readers not only witness to the importance of physical means for visualizing astronomical principles, but, as readers reproduce the most helpful figures, one can see this visual grammar in formation.

3. Modes of Calculation

What did these new collections of textual and visual commentaries on spherical astronomy mean for distinctively mathematical kinds of reading, beyond visualizing themselves walking around spheres? Did they encourage the reader to do more than simply repeat the qualitative doctrines of astronomy or basic geometry? Since books supplied more tools for calculation—tables and diagrams—students were confronted with the expectation that they engage in mathematical exercises. Readers also had other instruments and supplementary material available; one might imagine the reader with a map or globe in front of him. In a copy in the British Library, a reader doodled an image of a man holding up an armillary sphere for examination, while a book lies open on a stand beside him—perhaps a record of his own efforts to coordinate book and instrument.[122]

Many readers indeed performed calculations in the margins. The student notes of Beatus Rhenanus show how this might look. Although the majority of his annotations address qualitative questions, his opening notes also include astronomical calculations. His copy of Sacrobosco included, as many did, a short prefatory primer on geometrical shapes and—perhaps more importantly—on how to do sexagesimal arithmetic. Beyond simply working through the examples Lefèvre had provided, Beatus wrote down three extended paragraphs to describe, step by step, how to add and subtract degrees and minutes, and what to do with remainders.[123] Several of Beatus' notes go much

[121] Isabelle Pantin, 'Kepler's Epitome: New Images for an Innovative Book', in *Transmitting Knowledge: Words, Images, and Instruments in Early Modern Europe*, ed. Sachiko Kusukawa and Ian Maclean (Oxford, 2006), especially 229; Peter Barker and Kathleen M. Crowther, 'Training the Intelligent Eye', 448–451.

[122] Joannes Sacrobosco and Wenceslaus Faber de Budweys, *Opus Sphericum Ioannis de Sacro Busto Figuris et Perutili Commento Illustratum* (Cologne, 1508), BL IA:12221, a3r. Adam Mosley reflects on the role of instruments in pedagogy in 'Spheres and Texts on Spheres: The Book-Instrument Relationship and an Armillary Sphere in the Whipple Museum of the History of Science', in *The Whipple Museum of the History of Science: Instruments and Interpretations*, ed. Liba Taub and Frances Willmoth (Cambridge, 2006), 301–18.

[123] Lefèvre, *Textus de sphaera* (Paris, 1500), BHS K1046c, a2v. Other copies with similar notes include: Lefèvre,

further, reworking calculations that Lefèvre had done for his students in the commentary itself, which presented tables of the distances of the inner and outer surfaces of the orbs, their thicknesses, diameters, and finally the circumferences of these shells.[124] But whereas in the text Lefèvre did not show the operations used to gain these numbers, Beatus redid the calculations for each of the planets. For the circumference of the sun's orb, he wrote, 'the convex surface of the sun [i.e. the outer surface of the sun's shell], doubled, gives 7,930,000 miles, which when multiplied by 22 is 174,460,000, divided by seven makes 24933851, and a remainder of one tenth or one 9th'.[125] Then, after similarly describing the dimensions of the other planetary orbs, he continued with the most intriguing outcome, concerning how many degrees are subetended by a portion of each planetary orb: 'the circumference of the sun's heaven [i.e. its orb], which is 2,4[9]33,857, divided by 360 makes 69,23[0] with 57 remaining, in which place an integer is put in the book, that is 69,231'.[126] Here one can see how a degree grows in direct proportion to its distance from earth. Beatus simply narrates the operation in this verbal format, which means that he must have copied the results of long division or multiplication either from an abacus or tablet, or perhaps from a master performing the calculation in the lecture—small errors in the numbers, which do not affect the accuracy of the outcome, show that these notes are a 'clean copy', copied from previous work.[127]

Beatus therefore learned specifically mathematical kinds of reading practices through his study of Sacrobosco. The most important practice is familiar to any historian working with mathematical sources: reworking numbers on scratch paper. Redoing the work of Sacrobosco (or his commentator) gave Beatus the opportunity to see things he would have missed otherwise. The activity cements the kinds of operations (relationship of radius, circumference) involved in establishing basic dimensions of the known universe—affirming trust in those astronomical numbers. The activity of checking work allows Beatus to distance doing astronomy from the book alone. In his own words, he distinguishes his own calculations from what is 'in the book' (above). Further, doing the

---

Textus de sphaera (Paris, 1521), BnF res.v.209; Sacrobosco, *Libellus de sphaera* (Wittenberg, 1542), Houghton *GC.04625.474tm.

124 Lefèvre *Textus de sphaera* (Paris, 1500), a7r-v.

125 Beatus' Sacrobosco, a1v. 'Convexum solis duplatum reddit 7930000, qui multiplicatus per 22 [est] 174460000, et divisus per septem facit 24933851, et restat unus denarius sive una 9a.' N.B. that 7/22 was since antiquity the standard approximation of the ratio of a circle's diameter to its circumference.

126 Beatus' Sacrobosco, a1v. 'Dividendo circumferentiam celi solis que est 24833857 per 360 facit 6923, et restant 57 loco quorum ponutur integrum in libro scilicet 69231'.

127 E.g. in the example given just now, Beatus miscopied the value for the circumference, which is given in the text as 24**9**33857.

work himself also puts Beatus in a position to critique the book; his results leave remainders, which he observes have been smoothed into simple units in the book's tables.

This is not to render Beatus a thwarted mathematician. Rather, he exemplifies mathematical literacy developing within a humanistic context. Beatus' notes reveal him adopting these distinctively mathematical modes of reading—mathematical literacy—within a context of erudition. There is something of the philologist or grammarian's urge to gloss every line when Beatus writes a table for comparing units of measurement from the inch (*digitus*) to the mile. After all, it is hard to believe he actually intended to transmute the millions of miles between earth and the planets into cubits, feet, or palm-breadths. But a short note besides Lefèvre's account of measurements shows why Beatus found this worthy of note within astronomy. Astronomy measures the heavens and 'in the same way that a letter is the smallest part in grammar, and a word or locution is likewise the smallest part in logic, so also the inch is the smallest part in the discipline dealing with weight and measurements'.[128] At the Collège du Cardinal Lemoine, Beatus learned disciplines beginning with their principal, smallest parts.[129] Such encyclopedism, as several historians have shown, also motivated the grammarians, from Poliziano to Guillaume Budé, whose famous treatise *De asse et partibus* (Paris, 1514) showed the fundamental importance of understanding ancient coins, measurements, and their conversions for calculating the chronologies and texts of antiquity.[130] From this perspective, the future Beatus Rhenanus—the philologist so dear to Erasmus—is still visible in a table of unit conversions.

Glareanus was another example of such erudition—he updated Budé with his own *Liber de asse et partibus eius* (Basel, 1550)—but also hoped to make his students not only careful readers, but *mathematical* readers. On the same section of Sacrobosco that Beatus reworked, Glareanus also helped his students calculate the dimensions of the universe. Like Beatus he added a small table for converting units from inches to feet to miles. Then he addressed the problem of the earth's circumference. Following Eratosthenes, Sacrobosco suggests 252,000 stadia, which

[128] Beatus' Sacrobosco, a7r. 'Quemadmodum littera minimum quid est in grammatica, et vox seu dictio similiter minimum in logica, ita digitus minimum quid in ea disciplina que de mensuribus et ponderibus est'.
[129] On this aspect of Lefèvre's pedagogy, see Richard J. Oosterhoff, '*Idiotae* , Mathematics, and Artisans: The Untutored Mind and the Discovery of Nature in the Fabrist Circle', *Intellectual History Review*, (2014), 5–6.
[130] A relevant perspective on Poliziano's philological encyclopedism is found in Jean-Marc Mandosio, 'La classification des sciences et des arts à la Renaissance: Ange Politien, L'Omniscient (Panepistemon, 1492): édition, traduction et commentaire' (Doctoral Thesis, École Pratqiue des Hautes Études, 1998).

Glareanus observes is based on the proportion of one heavenly degree to 700 stadia. 'But, as [Gregor] Reisch says in the *Philosophical Pearl*, Ptolemy says that one degree corresponds to 500 stadia. Multiplied by the 360 degrees of heaven, these produce the number of 180,000 stadia' for the earth's diameter.[131] He went on to derive other basic values from 252,000 stadia for the earth's circumference, such as the earth's diameter and radius.

The student simply copying Glareanus' notes would have missed the distinctive mathematical skills Beatus learned by doing the calculations himself. With the benefit of a calculator, it becomes obvious that Glareanus has made some short cuts. For example, to determine the 'whole earth according to its three dimensions' he requires the multiplication of the earth's diameter (given as 80,181 stadia) by its circumference (252,000). But the product he gives is 20,205,864,000, which is the product of 80,182 stadia—because in fact he rounds a remainder down, at one point, and rounds it up, at another, to simplify the calculation. A student who missed this would have not learned to appreciate the ways small errors can creep into large calculations.

If less committed to raw calculation than Beatus' teacher, Glareanus nevertheless took extraordinary care to help his students visualize mathematical underpinning of physical phenomena. This is not only evident in the abundant diagrams he culled from various sources, but in his manuscript explanations themselves. He added a perspectival gloss to a section where Sacrobosco addressed the apparent magnification of the moon near the horizon, explaining it was caused by 'certain vapors rise between our sight and the sun … separating our visual rays'.[132] To understand this, Glareanus helped his reader through some geometry of perspective. In the margin he reconstructed a diagram to explain how of a light ray refracted through different media bends. 'The thing appears out of its actual location. So let cd be the surface of the water that contains the seen object b. Let there be a perpendicular bd erected, and an eye existing in the air at o. A ray proceeds from the visibile object to the eye [o], broken at point c by the perpendicular cf. By the 16th proposition of the *Perspectiva*, part 1, the ray continues to o; if the medium were uniform, it would proceed to g. Therefore you will see a pyramid between the object b and the perpendicular line bd joined at point l. And the same eye o judges [b] to be B.[133] So it is evident that the eye sees o in the water through a

[131] Glareanus' Sacrobosco, A4r. 'Ptolemaeus autem (ut auctor est Gaeorgius [*sic*] Reisch in Marg. Philoso.) uni gradui respondere dicit stadia 500, quae per gradus caeli 360 multiplicata illum stadiorum numerum producunt 180000'.

[132] Thorndike (ed.), *The Sphere of Sacrobosco*, 81.

[133] This is a puzzling identification—one would assume, from the diagram, that by 'B' Glareanus means l.

refracted ray (the basis for this is the 15th proposition of the *Perspectiva*, part 1). […] Thus an eye in a rarer medium sees an object in a denser medium as nearer and greater than it actually is'.[134] Twice Glareanus here cites the *Perspectiva communis* of John Peckham—yet neither proposition cited offers the geometrical argument of Glareanus, offering instead physical accounts of how denser mediums cause greater resistance to light rays.[135] Instead, Glareanus apparently gave his student his own geometrical reconstruction, extrapolating from one of the diagrams of a ray refracted in water (figure 9).

[insert figure here]

Figure 9. Glareanus' diagram of a light ray refracted in water in his copy of Sacrobosco, A3r (detail at left) bears resemblance to one in an early printed edition of John Peckham's *Perspectiva communis* (Venice, 1504), 4r (detail from Google Books at right). Note that I have added the 'b' where the figure has been trimmed off in rebinding.

The student copying this note would have encountered not a qualitative account of the reasons for refraction (which he could find in Peckham), but instead worked through a geometrical description, primarily in dialogue with a diagram. Specifically, the student must mentally track the relations of lines and points while keeping their definitions in view—a specifically geometrical form of reading.

Glareanus' attention to diagrams as a teaching tool was exemplary, in part because he recognized the limits of his medium. He repeatedly stated that concepts hard to see in the descriptions or diagrams in this book would be clearer with physical instruments. This was true for Sacrobosco's description of an instrument, where Glareanus said 'you do not understand what he says here about the astrolabe; but you will see, for it is easy to see this in [an actual] astrolabe'.[136] But Glareanus also pointed out the limitations of the two-dimensional diagrams he himself drew. The shape of the zodiac zones passing around the earth 'can be beautifully seen in a spherical body, and is hard to picture on a flat surface'.[137] The mind's eye, in imagining the heaven's motions, was limited by the challenge to the hand, in drawing on a plane. 'I think the reader will find this the hardest to draw', he wrote of the ascensions of the zodiacal

[134] Glareanus' Sacrobosco, A3r.
[135] John Peckham, *Perspectiva communis* (Venice, 1504), 4r.
[136] Glareanus' Sacrobosco, A4r. 'Hoc quod de Astrolabio hic dicit, non intelliges; in hoc videris, nam facile est hoc videre in astrolabio.'
[137] Glareanus' Sacrobosco, A5v. 'Et hoc pulchre in corpore sphaerica videri potest, et in plano depingi difficile'.

signs over the horizon. ‘For you will see this much more in a solid than in a plane. Therefore, so as not to waste time uselessly, I passed over these things’.[138] At some point, pen and paper fail to explain all the motions of the heavens. At that point, Glareanus uses his notes to gesture towards the world they are trying to understand. The page marks its own limits.

And marginalia in other copies suggest that the experience of Beatus and Glareanus was not isolated. An extreme example, showing the relationships between the expanding genre of introductions to spherical astronomy, can be found in one Houghton copy of Sacrobosco that is bound together with Fernel’s *Monalosphaerium* and *Cosmotheoria.* Fernel’s books focused on the geometrical and instrumental operations necessary to calculate days, distances, and planetary locations, but all three books in the volume are filled with scribbles in three hands, of which one in particular shows an obsession with calculation, filling margin after margin.[139]

[insert figure here]

Figure 10. Lefèvre, *Textus de sphaera* (Paris, 1527), Houghton f EC.Sa147s.1527, title page.

As enthusiastic as this reader was, his mathematical skills were not terribly sophisticated—most scribblings appear to be to calculate the periods of planets to find planetary conjunctions—but nevertheless these fervid sums indicate readers increasingly turning to quantification even in introductory works.

Many readers did little more than label the diagrams already printed in their Sacrobosco; others filled the diagrams in and decorated them with flourishes.[140] Among the most interesting, as like those already discussed in this paper, are the horoscopes drawn in the margins of such textbooks—particularly in volumes that contain Lefèvre’s commentary. Even though Lefèvre and his students (such as Josse Clichtove) warned that they did not teach astronomy primarily for prognostication, they offered examples of how to locate planets and constellations.

[138] Glareanus’ Sacrobosco, B2r. ‘Lector probare hanc depingere difficillimum puto. Longe enim in solido magis quam in plano haec conspicies. Ne igitur tempus inutiliter confuseremus, consulto missa faecimus’. At the bottom of the same page he similarly promised that turning to a three-dimensional model would make the example plain: ‘Haec si luculentius videre velis in rectu habes in solide corpore conspicere, quoniam in planum figere hoc laboriosum invenies’ (If you want to see this more clearly, regarding the right [ascension], you should look at a solid sphere, because you will find this more difficult to set up here, on a plane).

[139] Houghton f EC.Sa147s.1527, a *Sammelband* comprising Lefèvre, *Textus de sphera* (Paris, 1527); Jean Fernel, *Monalosphaerium, partibus constans quatuor* (Paris, 1527); Jean Fernel, *Cosmotheoria, libros duos complexa* (Paris, 1528).

[140] Many diagrams are clarified with short descriptions: e.g. Newberry VAULT Ayer QB41 .S12 1508 no. 1; BL 417.d.4.

This was enough for some readers, who analyzed horoscopes in the very margins of their copies.[141]

[insert figure 6 here]

Figure 6. Compendium of the *Textus sphaerae* (Venice, 1508), Huntington, Burndy 751765, 55v. Such a detailed horoscope would have required the aid of other reference works, notably an ephemerides.

Quite possibly such horoscopes originated in collections of genitures later published in the sixteenth century by erudite experts such as Girolamo Cardano or Luca Guarico.[142] Certainly these markings reveal readers integrating their basic textbooks with the growing field of technical knowledge—interpreting the qualitative description of astronomical principles in Sacrobosco as well as reflecting on the quantitative information in the charts of planetary positions. These traces remind us that, through the practice of astrology, astronomy served physicians and court advisers throughout Renaissance Europe—and as Sacrobosco's apparatus grew more technically sophisticated, so did its utility to prognosticators, whether would-be professionals or curious do-it-yourselfers.

Such notes suggest that technical discussions in textbooks did, in fact, make technicians. Together, Beatus and Glareanus exemplify a mathematical literacy often taken for granted in the history of mathematics, even though this growing literacy links the new technical texts pouring from the printing presses with the growing fascination with mathematical learning in the sixteenth century.[143] In particular, these erudite humanists-in-training exemplify the kind of basic skills needed to read the newly-fashionable cosmographies; both readers encountered Sacrobosco in dialogue with either Ptolemy's *Cosmographia* or Ringmann's *Cosmographiae introductio*. I would suggest that by watching Beatus and Glareanus learn and teach, we see mathematical literacy become an integral part of erudite education—and that this included some distinctively mathematical forms of reading, such as reworking calculations,

141 E.g. BnF res.v.209 (Paris, 1521), 13v; Huntington, Burndy 751765 (Venice, 1508), fol. 55v; BL 8562.f.34 (Paris, 1534), fol. 29r.

142 On the genre, see Anthony T. Grafton, "Geniture Collections, Origins and Uses of a Genre," in Frasca-Spada and Nicholas Jardine (eds.) *Books and the Sciences in History*, 49–68. See also Nicholas Popper, 'The English Polydaedali: How Gabriel Harvey Read Late Tudor London', *Journal of the History of Ideas* 66 (2005), 351-381.

143 A classic description of the growing technical literature is Elizabeth Eisenstein, *The Printing Press as an Agent of Change: Communications and Cultural Transformations in Early-Modern Europe* (Cambridge, 1979), 520–575. Some sense of the growing genre can be seen in the bibliography assembled in Eva G. R. Taylor, *The Mathematical Practitioners of Tudor & Stuart England* (Cambridge, 1954). Printed technical books are only the most visible portion of a rich culture of mathematical practitioners, as argued by Stephen Johnston, 'The Identity of the Mathematical Practitioner in 16th-Century England', in *Der 'mathematicus': Zur Entwicklung Und Bedeutung Einer Neuen Berufsgruppe in Der Zeit Gerhard Mercators*, 93–120.

relating diagrams to geometrical narratives, and indeed endeavoring to mentally reconstruct three-dimensional movements from words and limited diagrams.

Conclusion

Reader marks from this long period suggest that as early modern textbooks diversified in print, Sacrobosco was dressed up in ever richer layers of images, commentary, and technical apparatus, and the book's audience grew more literate in the arts of quantification. The point is significant, precisely because Sacrobosco was the most basic of introductions to astronomy. If mastering Sacrobosco now required more than an intuitive sense for geometry and also entailed some knowledge about the calculation of longitudes and latitudes, then the early sixteenth century is an important locus in the shifting technical literacy of Europe—to wax grandiloquent, a weighty moment in the history of the interface between practice and theory. Readers working in margins witness to this shift.

It is beyond the scope of this article to claim whether the changes described in the first part of the paper—the expanding collections of commentaries and visual programs of Sacrobosco—in fact *caused* changes in reading habits. A careful argument for this claim would investigate annotations and marginalia in manuscripts, and more rigorously compare earlier to later printed editions. But I can report that the most technically sophisticated annotations are usually found the margins of books with denser commentary and visual apparatus. Beatus' notes show this well. Glareanus' Sacrobosco is unusual as an example of dense, technically sophisticated annotations appearing in an edition devoid of images—but Glareanus, as I have shown, depended on several visually sophisticated editions of Sacrobosco in order to create his unique working copy. Based on the evidence so far, I would contend, books with more technical apparatus and tables were likely to foster greater technical literacy.

But this legacy is deeply ambiguous. Certainly such well-appointed textbooks firmly ensconced the mathematical disciplines within the training of elites, even encouraging practical uses of that mathematics—at least in some students. But their annotations rarely indicate a program of mathematical invention. For most, the regime seems to be one of memorizing rules, applying them, and moving on to the next topic. One need not go so far as Lisa Jardine and Anthony Grafton in their celebrated account of humanist education as docility-inducing drudgery to observe that readers rarely found in Sacrobosco the spark of mathematical ingenuity or creativity.[144] The controversial Gabriel Harvey recorded mostly gossip about the 'best' mathematical authors and propaganda for the practical significance of the subject, in his copy of Sacrobosco.[145] Perhaps Sacrobosco, diverse a genre as it had become, remained the wrong sort of book for deeper creativity.

At the same time, the fact that students were reading and doing more, and more sophisticated, astronomy should not obscure the fact that the cosmology Sacrobosco taught did not change. Earlier I described a copy of Sacrobosco now at Houghton Library as an extreme example of how readers increasingly tended to work out astronomy in calculation. As part of its frontispiece is the phrase 'Altius insurgit animus sub imaginem mundi' (The mind rises higher below the figure of the world).[146] Ptolemy and Sacrobosco might have subscribed to this ancient lofty goal for astronomy, and certainly Lefèvre claimed that mathematics was primarily important for his students because it would raise their souls to the divine.[147] When seen in the context of the other scribbles and scratches that fill the margins of the volume, this phrase becomes an instructive paradox. The high religious aims of natural inquiry were not an alternative to or disconnected from the increasingly practical and technical applications of mathematical inquiry. Readers could hold these different aims together. The broadest conclusion of this paper, then, is that early modern astronomical books were integral to the growing emphasis on mathematical utility[148]—and yet, by incorporating the latest insights into an ancient genre, they also fostered a profound conservativism that embedded the old world picture deeply into early modern culture.

Reader marks make clear that Sacrobosco's *Sphere* remained important for a long time. I have concentrated on sixteenth-century readers—but marginalia offer another insight into the length of time that Sacrobosco served as an introduction to an understanding of the cosmos. While the book was printed through the middle of the

---

[144] Anthony Grafton and Lisa Jardine, *From Humanism to the Humanities: Education and the Liberal Arts in Fifteenth- and Sixteenth-Century Europe* (Cambridge, MA, 1986).

[145] Lefèvre, *Textus de Sphaera* (Paris, 1527), BL 533.k.1.

[146] Lefèvre, *Textus de Sphaera* (Paris, 1527), Houghton f EC.Sa147s.1527, titlepage.

[147] Cf. Lefèvre's prefatory letter to his Astronomicon, a contribution to the theoric tradition: Rice, *Prefatory Epistles*, 112–4.

[148] On the language of utility to support the rising prestige of mathematics, see Katherine Neal, 'The Rhetoric of Utility: Avoiding Occult Associations for Mathematics Through Profitability and Pleasure', *History of Science* 37 (1999), 151–78; Lesley B. Cormack, 'The Commerce of Utility: Teaching Mathematical Geography in Early Modern England', *Science & Education* 15 (2006), 305–22.

seventeenth century, the last Elzevier edition of 1656 has been considered ‘anachronistic’.[149] Perhaps. But such a judgment does not account for the saturation of seventeenth-century libraries with volumes that were already old and yet were read afresh. The Houghton volume just mentioned bears on the titlepage ‘perlegitur Mart. 1, 1646/7’, and it is only one of a great many sixteenth-century editions that bear the marks of seventeenth-century readers. If we wish to understand why the geocentric world picture captured the imaginations of most Europeans until well into the seventeenth century, understanding Sacrobosco’s readership is a good start.

Centre for Research in the Arts, Social Sciences and Humanities

Alison Richard Building, 7 West Road, Cambridge, CB3 9DT

[149] Gingerich, ‘Sacrobosco Illustrated’, 211.